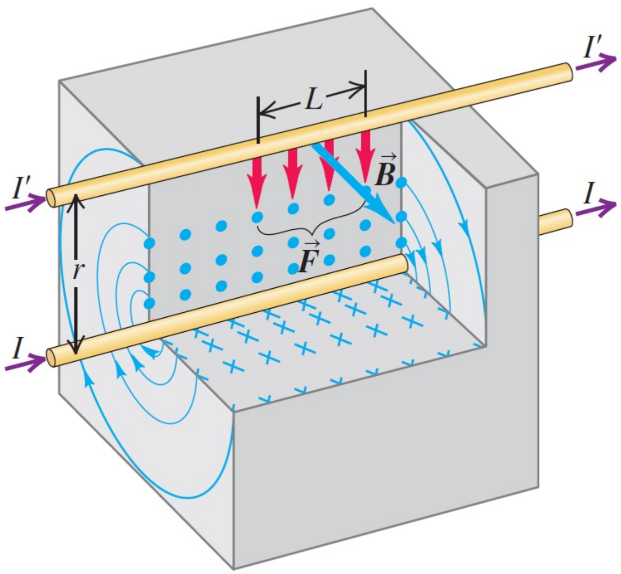
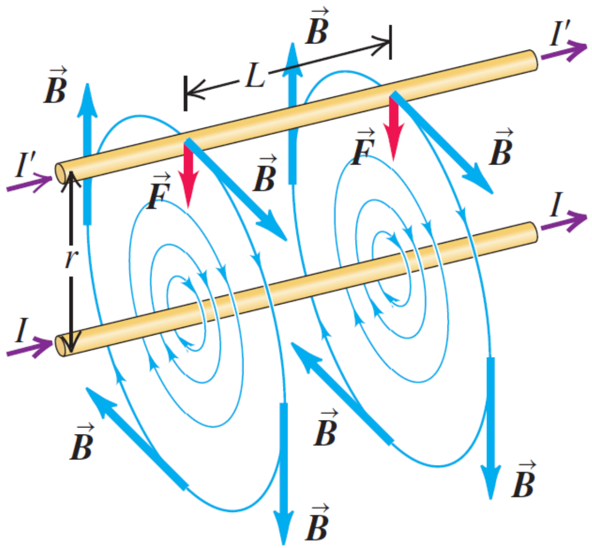
ELECTRICITY AND MAGNETISM (PHYS 231)

Lecture 18: Sources of Magnetic Field

Oct 30, 2024

Force between Parallel Conductors

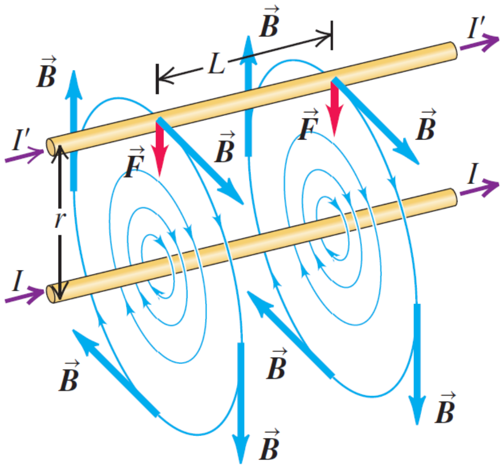




- Parallel conductors carrying current in the same direction attract each other.
- Parallel conductors carrying current in the opposite directions repel each other.

Right-Hand Rules!

Force between Parallel Conductors



$$B = \frac{\mu_0 I}{2\pi r}$$

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$$

Magnetic force per unit length between two long, straight, parallel, current-carrying conductors

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

- μ_0: Magnetic constant
- I: Current in first conductor
- I': Current in second conductor
- r: Distance between conductors

- Parallel conductors carrying current in the **same** direction **attract** each other.
- Parallel conductors carrying current in the **opposite** directions **repel** each other.

Example

Two long, parallel wires are separated by a distance of 2 cm. The force per unit length that each wire exerts on the other is 3×10^{-5} N/m, and the wires repel each other. The current in one wire is 0.600 A.

What is the current in the second wire?

Are the two currents in the same direction or in opposite directions?

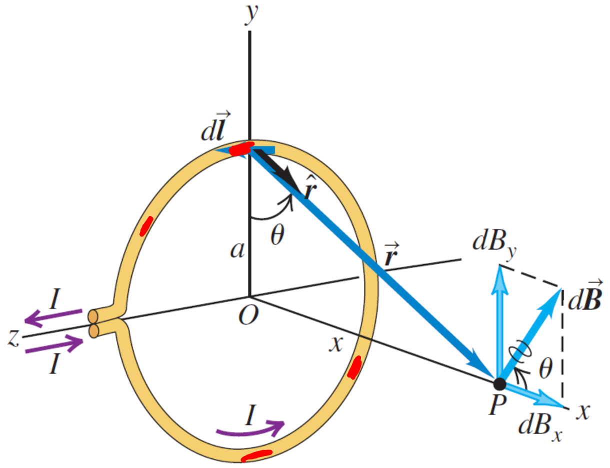
$$3\times10^{-5}\,\text{N/m} = \frac{F}{L} = \frac{\mu_0}{2\pi r} I I' = \frac{4\pi\times10^{-7}}{2\pi\times10^{-2}\times2}\, 6\times10^{-1}\, I' = 6\times10^{-6}\, I'$$

($10^{-2}\times2$ = 2 cm)

$$\Rightarrow \quad I' = \frac{3\times10^{-5}}{6\times10^{-6}} = \frac{30}{6} = 5\,\text{A}$$

Circular Current Loop (Sec. 28.5)

Find the magnetic field at a point P on the axis of the loop, at a distance x from the center.



Step 1 Find $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{a}{(x^2+a^2)^{1/2}}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{x}{(x^2+a^2)^{1/2}}$$

Step 2 $\vec{B} = \int d\vec{B}$

$B_y = 0$ (symmetry argument)

Magnetic field on axis of a circular current-carrying loop

$$B_x = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

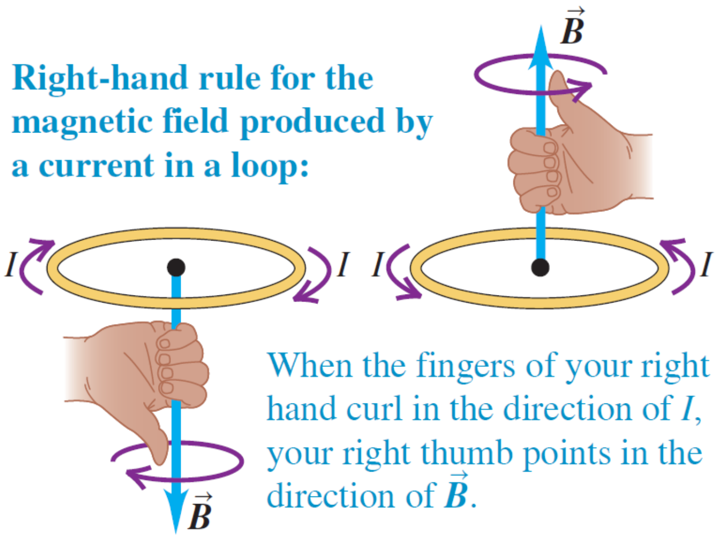
Magnetic constant · Current · Radius of loop · Distance along axis from center of loop to field point

Circular Current Loop (Sec. 28.5)

Magnetic field on axis of a circular current-carrying loop

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

Magnetic constant; Current; Radius of loop; Distance along axis from center of loop to field point



$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$$

N # of current loops (i.e. a solenoid)

$x = 0$

Magnetic field at center of N circular current-carrying loops

$$B_x = \frac{\mu_0 N I}{2a}$$

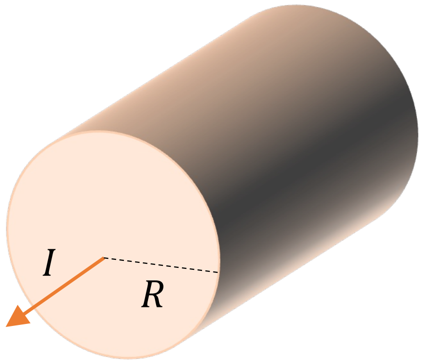
Magnetic constant; Number of loops; Current; Radius of loop

Magnetic Field of a Long Cylindrical Conductor

Find the magnetic field generated by a infinitely long cylindrical conductor with current I.

a) At $r < R$.

b) At $r > R$.



Using Biot-Savart Law?

Yes, but we will get the answer one hour later…

That's why we need **Ampere** to save us!

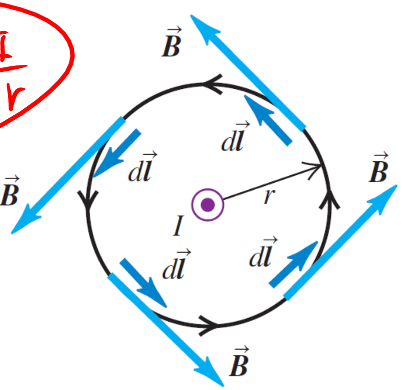
Ampere's Law

Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

- Line integral around a closed path: $\oint$
- Scalar product of magnetic field and vector segment of path: $\vec{B} \cdot d\vec{l}$
- Magnetic constant: μ_0
- Net current enclosed by path: I_{encl}

Ampere's Law

The line integral of magnetic field around a closed path equals to μ_0 times the total current enclosed by the path.

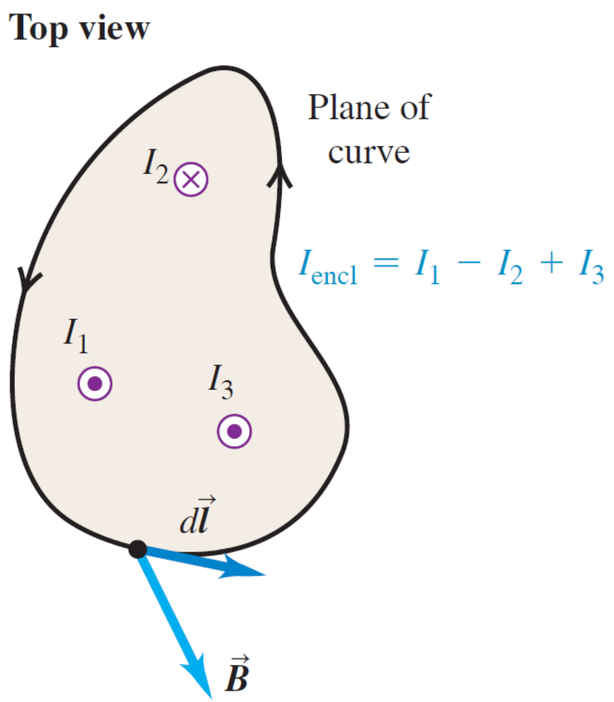


Example

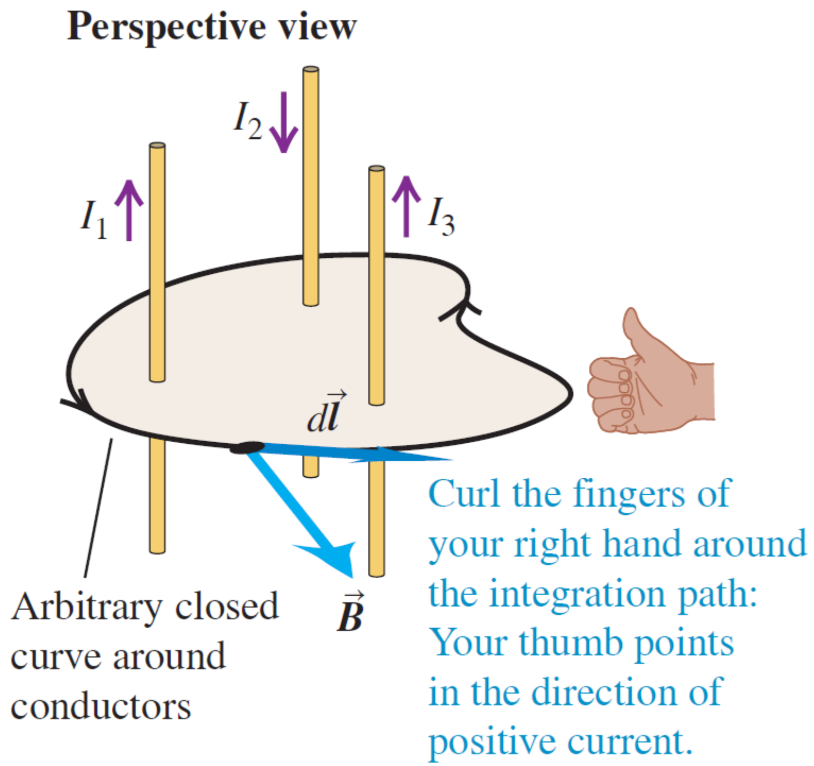
A long straight wire generate a magnetic field $B = \frac{\mu_0 I}{2\pi r}$ around it (following the right-hand rule)

$$\oint \vec{B} \cdot d\vec{l} = \int_0^{2\pi r} \frac{\mu_0 I}{2\pi r} dl = \frac{\mu_0 I}{2\pi r} \times 2\pi r = \mu_0 I$$

Ampere’s Law



Ampere’s law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.



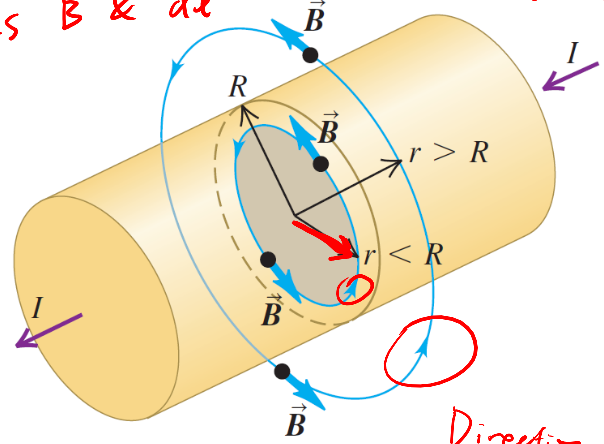
Magnetic Field of a Long Cylindrical Conductor (Example 28.8)

Find the magnetic field generated by a infinitely long cylindrical conductor with current I.

a) At $r < R$.

b) At $r > R$.

$$\oint \vec{B} \cdot d\vec{l} = B\, 2\pi r$$ (assumes $\vec{B}$ & $d\vec{l}$ in the same direction)



b) Choose a loop with radius $r > R$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$= B \cdot 2\pi r$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r} \quad (\text{for } r > R)$$

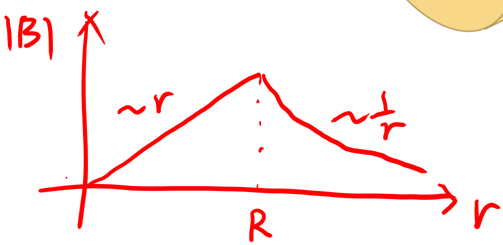
a) Current density $J = \dfrac{I}{\pi R^2}$

$$I_{enc} = J \cdot \pi r^2 = I \frac{r^2}{R^2}$$

$$\mu_0 I_{enc} = B \cdot 2\pi r$$

$$= \mu_0 I \cdot \frac{r^2}{R^2}$$

$$\Rightarrow B = \mu_0 \frac{I r}{2\pi R^2} \quad (\text{for } r < R)$$



Direction of loop

⇓ RHR

I is positive

⇓

$B > 0 \Rightarrow \vec{B}$ follows the direction of the loop.

Magnetic Field of a Solenoid (Example 28.9)

Find the field at or near the center of a solenoid if it has n turns per unit length and carries current I.

1. Choose a proper path direction.
2. Perform the loop integral.

$$\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\, dl + \underbrace{\int_b^c \cdots + \int_c^d \cdots + \int_d^a \cdots}_{B=0}$$

$$\mu_0 I_{enc} = B \cdot L$$

$$\mu_0 \cdot n L I \Rightarrow \boxed{B = \mu_0 n I}$$