ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 17: Sources of Magnetic Field

Oct 28, 2024

Magnetic Field

Electric Field	Magnetic Field
Origin: Electric charges	Origin: **Moving** charges or current

Electric field & magnetic field are the two sides of the same coin! They can be transformed to each other via a reference of frame transformation.

Magnetic Field of a Moving Charge

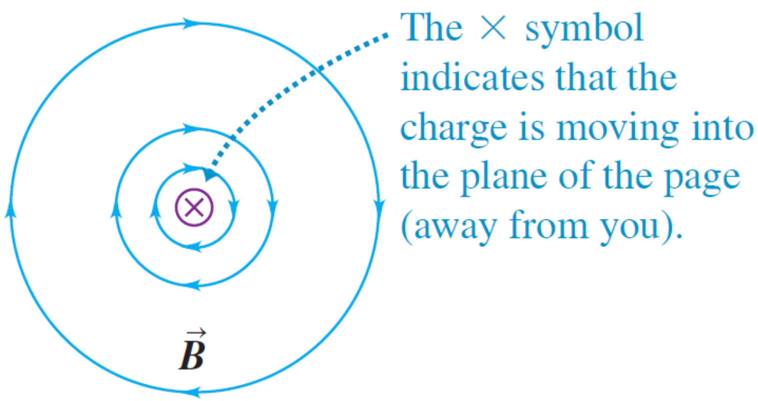
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Magnetic field due to a point charge with constant velocity

Magnetic constant: μ_0; Charge: q; Velocity: $\vec{v}$; Unit vector from point charge toward where field is measured: $\hat{r}$; Distance from point charge to where field is measured: r

$$\mu_0 = 4\pi\times10^{-7}T\cdot m/A$$

(b) View from behind the charge

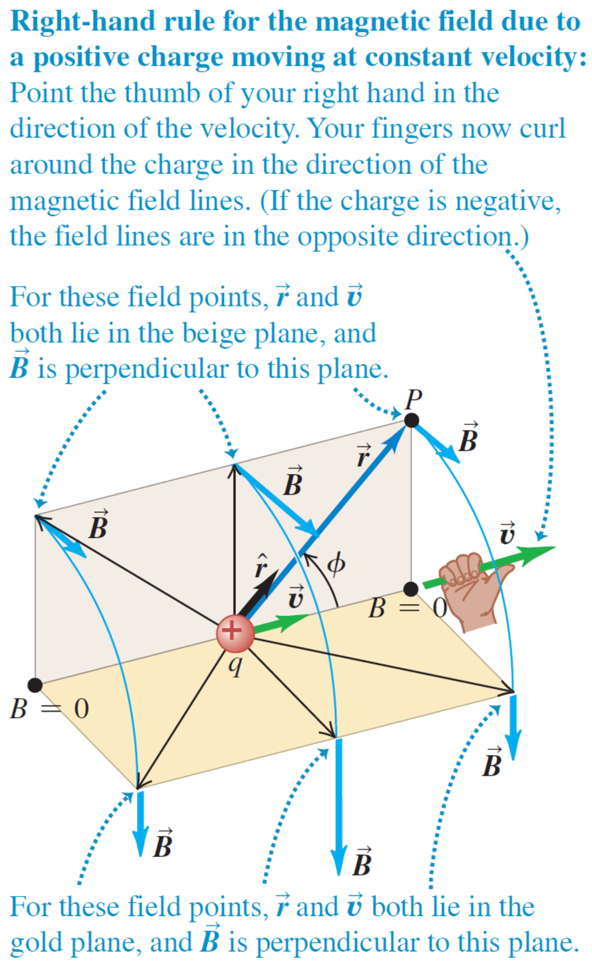


Positive charge

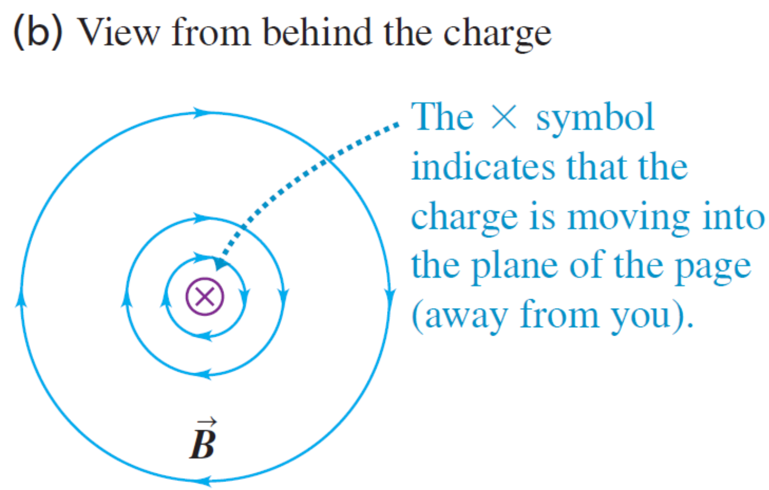
$$\frac{1}{\sqrt{\varepsilon_0\mu_0}} = 2.998\times10^8 m/s$$

Speed of light in vacuum!

Velocity Selector



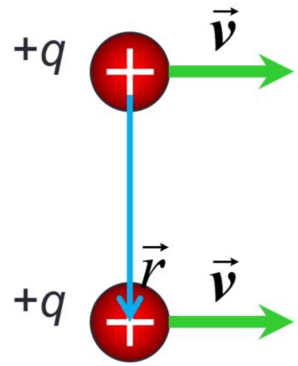
$$|B| = \frac{\mu_0}{4\pi} \frac{|q| v \, sin\phi}{r^2}$$ Magnitude of $\vec{B}$



Positive charge

Clicker question

Two positive point charges move side by side in the same direction with the same constant velocity. What is the direction of the magnetic force that the upper point charge exerts on the lower one?



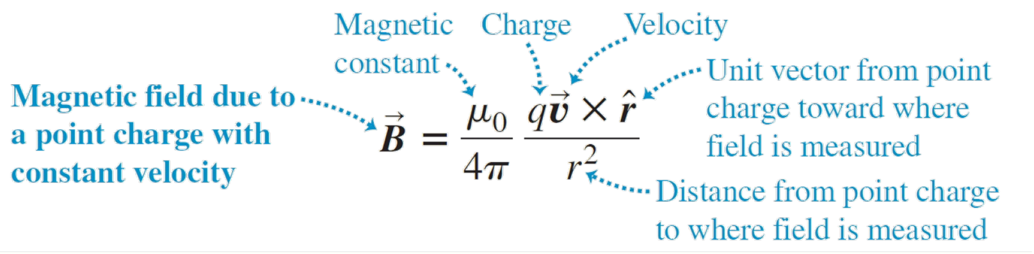
A. toward the upper point charge (the force is attractive)

B. away from the upper point charge (the force is repulsive)

C. in the direction of the velocity

D. opposite to the direction of the velocity

E. none of the above



Current-induced Magnetic Field

Principle of Superposition of Magnetic Field

The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.

Consider a short segment $d\vec{l}$ of a current-carrying conductor. Cross-section area is A, charge density is n. The total moving charge

$$dQ = nqAdl$$

Assuming drift velocity v_d,

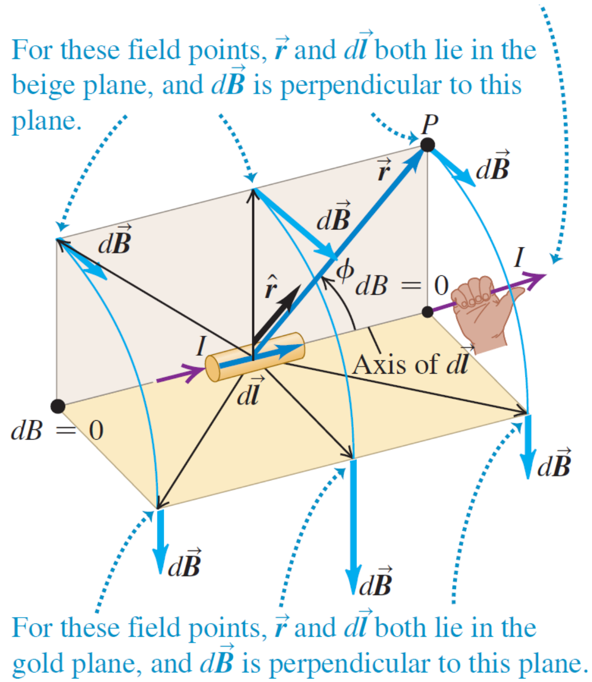
$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(n|q|Av_d)\ dlsin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\ dl\ sin\phi}{r^2}$$

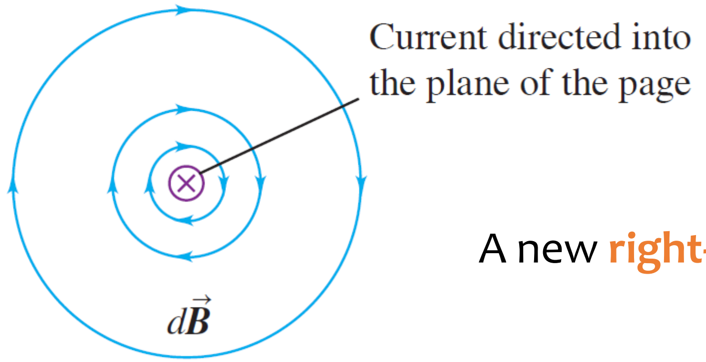
Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

- **Magnetic field due to an infinitesimal current element**: $d\vec{B}$
- Magnetic constant: μ_0
- Current: I
- Vector length of element (points in current direction): $d\vec{l}$
- Unit vector from element toward where field is measured: $\hat{r}$
- Distance from element to where field is measured: r

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l} \times \hat{r}}{r^2}$$





A new **right-hand rule** for current-induced magnetic field.

Example

A copper wire carries a steady 125 A current. Find the magnetic field due to a 1.0 cm segment of this wire at a point 1.2 m away from it, if the point is (a) point P1, straight out to the side of the segment, and (b) point P2, in the xy-plane and on a line at 30 degree to the segment.

$\hat{i} \times \hat{i} = 0 \qquad \hat{i} \times \hat{j} = \hat{k}$

(a) P_1: $d\vec{l} = 1\,cm \times (-\hat{i})$

$\vec{r} = 1.2\,m\,(\hat{j})$

$|r| = 1.2\,m, \quad \hat{r} = \hat{j}$

$\vec{B} = (1.0\times10^{-7}\,T\cdot m/A)\,\dfrac{125\,A\;10^{-2}\,m(-\hat{i})\times\hat{j}}{(1.2\,m)^2}$

$= (\ldots)(-\hat{k})$ #

P_2: $d\vec{l} = 10^{-2}\,m \times (-\hat{i})$

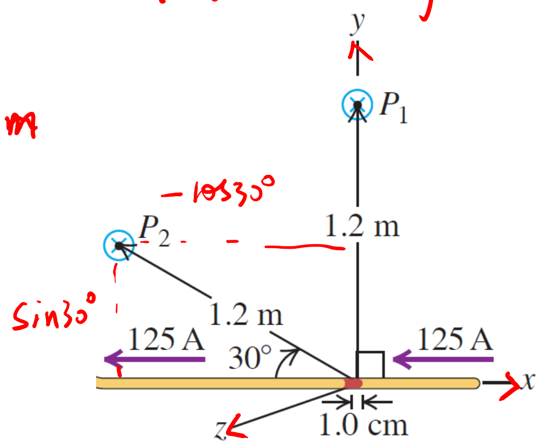
$\vec{r} = (-\cos30^\circ\,\hat{i} + \sin30^\circ\,\hat{j})\;1.2\,m$

$\hat{r} = (-\cos30^\circ\,\hat{i} + \sin30^\circ\,\hat{j})$

$|r| = 1.2\,m$

$\vec{B} = (1.0\times10^{-7}\,T\cdot m/A)\,\dfrac{125\,A\times10^{-2}\,m\,(-\hat{i})\times(-\cos30^\circ\,\hat{i} + \sin30^\circ\,\hat{j})}{(1.2\,m)^2}$

$= (\ldots)(-\hat{k})$

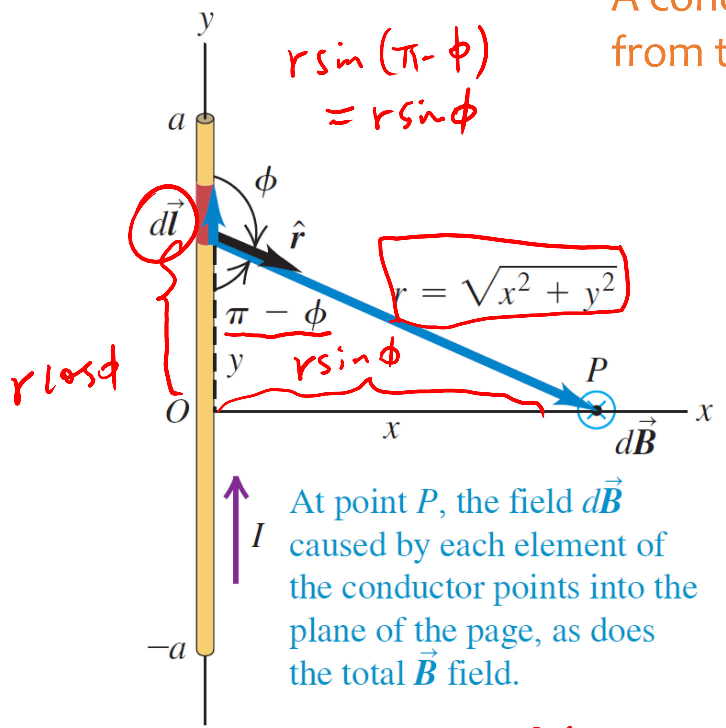


$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

$$\mu_0 = 4\pi\times10^{-7}\,T\cdot m/A$$

A Long Straight Wire

A conductor with length $2a$ carries a current I. Find $\vec{B}$ at a point a distance x from the conductor on its perpendicular bisector.



At point P, the field $d\vec{B}$ caused by each element of the conductor points into the plane of the page, as does the total $\vec{B}$ field.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} =$$

$$d\vec{l} = dy\,\hat{j}$$

$$\vec{r} = r(\sin\phi, \cos\phi) = r\sin\phi\,\hat{i} + r\cos\phi\,\hat{j} \Rightarrow \hat{r} = \sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x\,dy}{(x^2+y^2)^{3/2}} \Rightarrow B = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}$$

$$\sin\phi = \frac{x}{\sqrt{x^2+y^2}} \qquad \cos\phi = \frac{y}{\sqrt{x^2+y^2}}$$

$$a \to \infty \qquad \sqrt{x^2+a^2} \approx a$$

$$B = \frac{\mu_0 I}{4\pi}\frac{2}{x} = \frac{\mu_0 I}{2\pi x}$$

An Infinitely Long Straight Wire

A conductor with length $2a$ carries a current I. Find $\vec{B}$ at a point a distance x from the conductor on its perpendicular bisector.

Magnetic field near a long, straight, current-carrying conductor

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic constant (μ_0); Current (I); Distance from conductor (r)

Q: An infinitely long, straight conductor carries a 1.0 A current. At what distance from the axis of the conductor does the resulting magnetic field have magnitude $0.5 * 10^{-4}$ T?

$$0.5\times10^{-4}\,T = B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\times10^{-7}}{2\pi}\frac{1}{r} \Rightarrow r = \frac{4\pi\times10^{-7}}{2\pi}\frac{1}{0.5\times10^{-4}}$$

$$= 4\times10^{-3}\,m = 4\,mm.$$

Two long, straight wires are oriented perpendicular to the xy-plane. They carry currents of equal magnitude I in opposite directions as shown. At point P, the magnetic field due to these currents is in

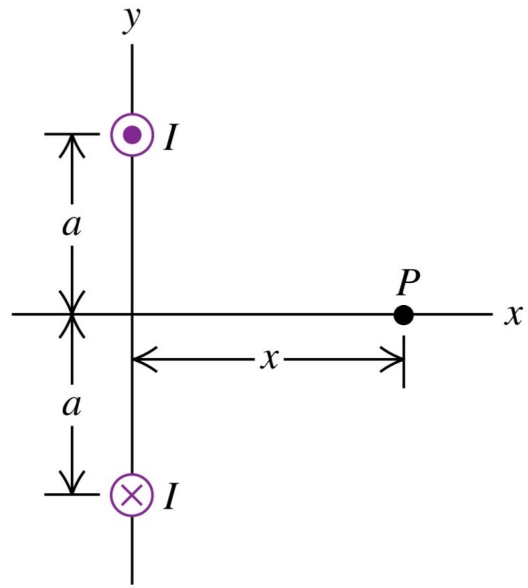
✓ A. the positive *x*-direction.

B. the negative *x*-direction.

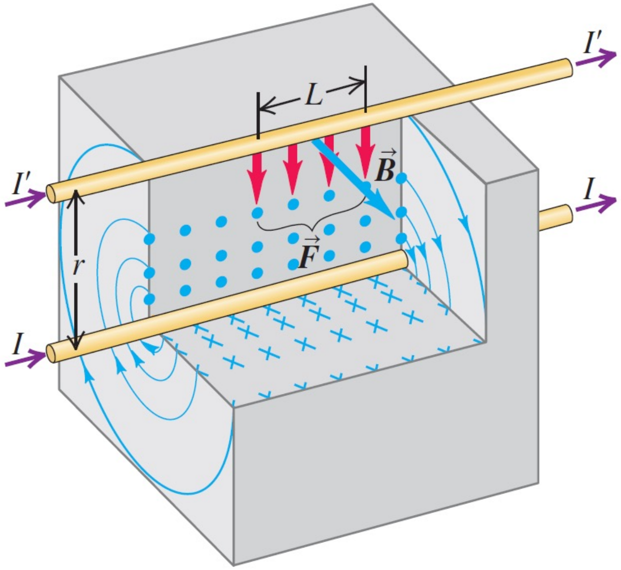
C. the positive *y*-direction.

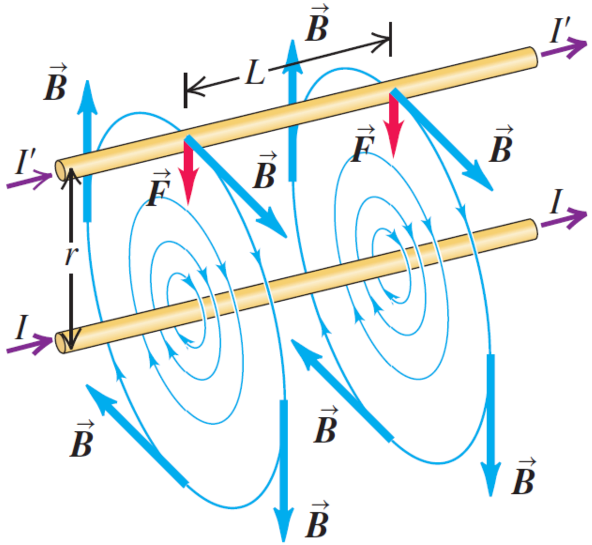
D. the negative *y*-direction.

E. None of the above



Force between Parallel Conductors

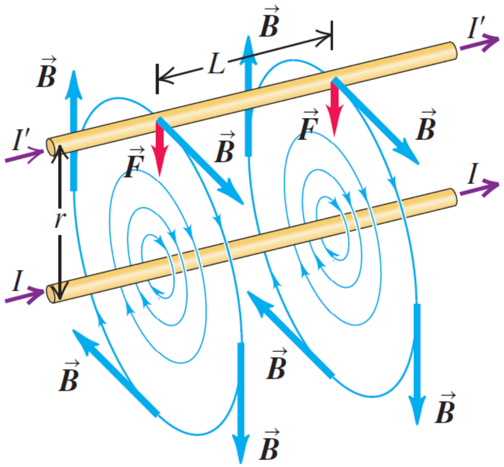




- Parallel conductors carrying current in the same direction attract each other.
- Parallel conductors carrying current in the opposite directions repel each other.

Right-Hand Rules!

Force between Parallel Conductors



$$B = \frac{\mu_0 I}{2\pi r}$$

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$$

Magnetic force per unit length between two long, straight, parallel, current-carrying conductors

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

Magnetic constant (μ_0)
Current in first conductor (I)
Current in second conductor (I')
Distance between conductors (r)

- Parallel conductors carrying current in the same direction attract each other.
- Parallel conductors carrying current in the opposite directions repel each other.