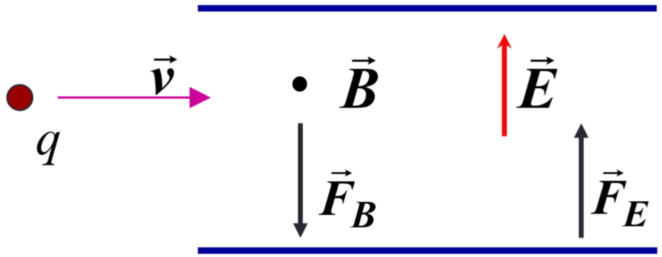
ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 16: Magnetic Field & Magnetic Forces

Oct 23, 2024

Velocity Selector

If a charged particle q is experiencing both $\vec{E}$ and $\vec{B}$: $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$



If $q > 0$, then the magnetic force will be downwards and the electric force will be upwards.

It is possible to achieve a perfect cancellation between magnetic & electric forces.

$$qE = qvB \quad \Rightarrow \quad v = \frac{E}{B}$$

Velocity selector: only particles with the right velocity can pass!

A charged particle moves through a region of space that has both a uniform electric field and a uniform magnetic field. In order for the particle to move through this region at a constant velocity,

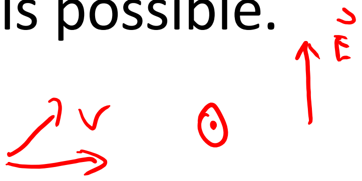
A. the electric and magnetic fields must point in the same direction.

B. the electric and magnetic fields must point in opposite directions.

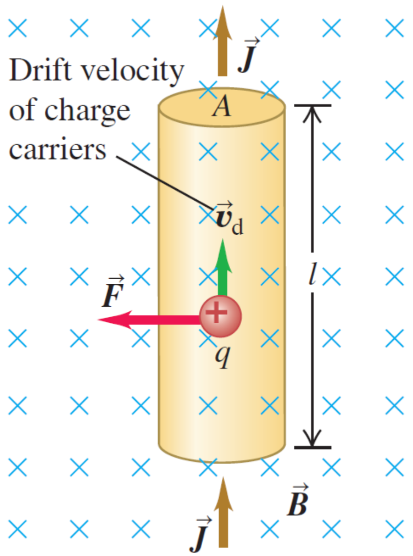
C. the electric and magnetic fields must point in perpendicular directions.

D. more than one of A, B, or C is possible.

[✓ E. not enough information]



Magnetic Force on a Current-Carrying Wire



We start with a wire of length l and cross section area A in a magnetic field of strength B with the charges having a drift velocity of v_d.

If the charge density is n, the total charge in this wire is nAl.

The magnetic force on a single charge is qv_dB.

The total force on the wire is $F = qv_dBnAl = \boxed{(nqv_d)A}lB$

$$I = JA = \boxed{(nqv_d)A} \qquad \underline{F = IlB}$$

Magnetic force on a straight wire segment $\vec{F} = I\vec{l} \times \vec{B}$

Current

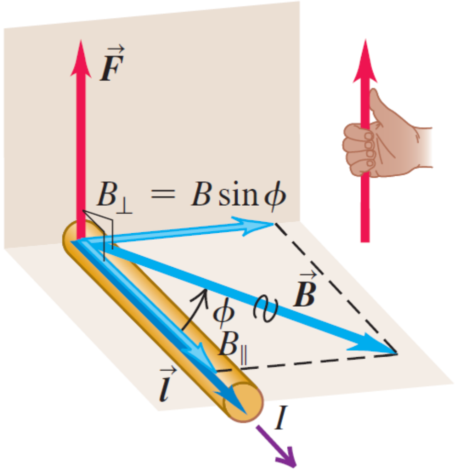
Magnetic field

Vector length of segment (points in current direction)

Magnetic Field

Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

- Magnitude is $F = IlB_{\perp} = IlB\sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.



Current

Magnetic force on a straight wire segment $\vec{F} = I\vec{l} \times \vec{B}$ Magnetic field

Vector length of segment (points in current direction)

○ The force is **ALWAYS** perpendicular to both the wire & the magnetic field.

○ The direction of $\vec{F}$ is determined by the **right-hand** rule.

Warning: Use your right hand even if you are left-handed.

Example

A wire 20.0 cm long lies along the z-axis and carries a current of 5 A in the +z-direction. The magnetic field is uniform and has components Bx = -0.2 T , By = -0.9 T , and Bz = -0.3 T.

Find the magnetic force on the wire and its magnitude.

$\vec{F} = I\vec{l} \times \vec{B}$ $\vec{l} = 0.2m\,\hat{k}$ $\vec{B} = -0.2\hat{i} - 0.9\hat{j} - 0.3\hat{k}$

$$= (5A \cdot 0.2m)\,\hat{k} \times (-0.2\hat{i} - 0.9\hat{j} - 0.3\hat{k})T$$

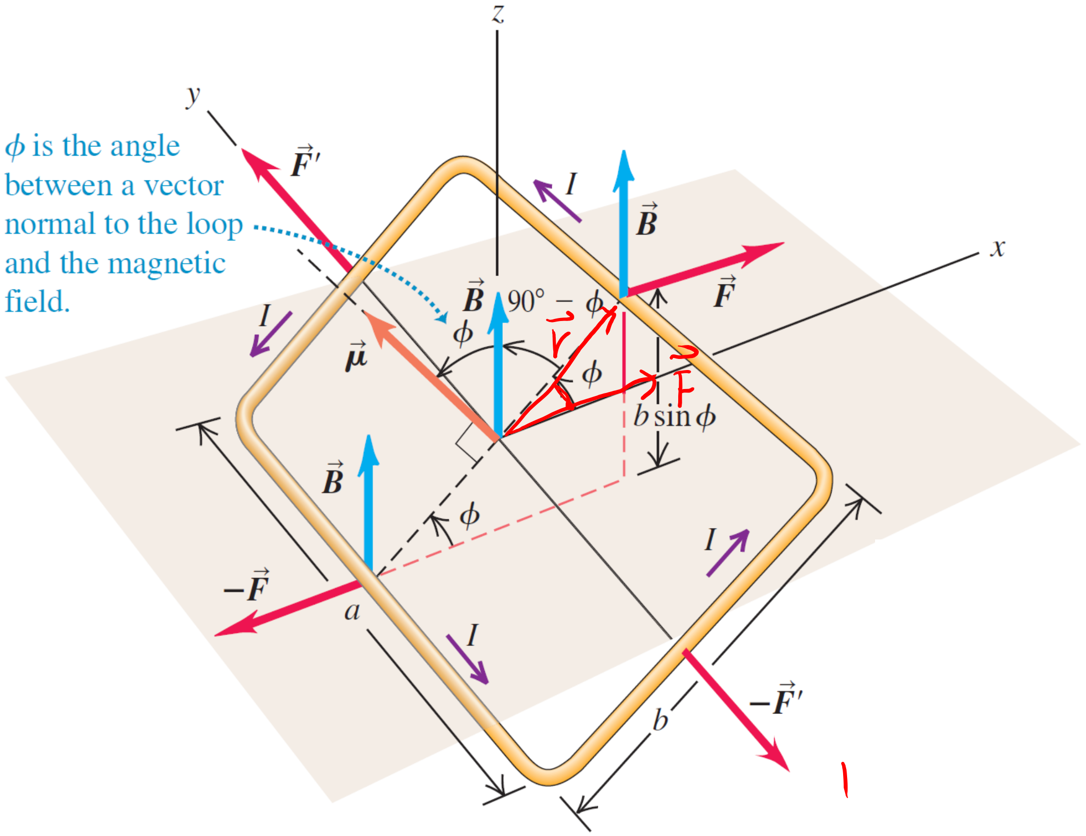
$$= (1\,A \cdot m)(-0.2\hat{j} + 0.9\hat{i})T$$

$$= (+0.9\hat{i} - 0.2\hat{j})(A \cdot m)T$$

$$\left\{\begin{array}{l} \hat{k} \times \hat{i} = \hat{j} \\ \hat{k} \times \hat{j} = -\hat{i} \\ \hat{k} \times \hat{k} = 0 \end{array}\right.$$

$$|\vec{F}| = \sqrt{(0.9)^2 + (0.2)^2} = \sqrt{0.81 + 0.04} = \cdots$$

Current Loop in a Magnetic Field



ϕ is the angle between a vector normal to the loop and the magnetic field.

The **net force** on a current loop in a uniform magnetic field is **zero**.

However, the **net torque** is **not** in general equal to zero.

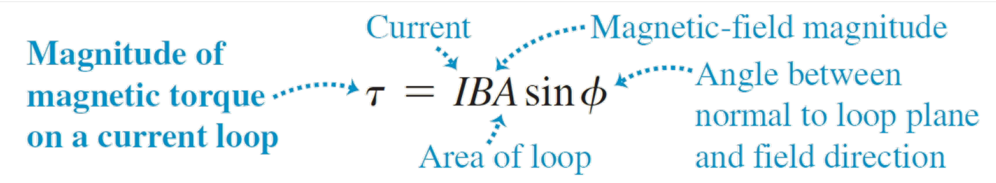
$$F = IaB, \quad F' = IbBcos\phi$$

Torque: $\underline{\vec{\tau} = \vec{r} \times \vec{F} = rFsin\phi}$

The **lever arm** for each force is $\frac{b}{2}sin\phi$

The net torque is $\tau = 2F\left(\frac{b}{2}\right)sin\phi = \underline{IBab\ sin\phi}$

Magnetic Dipole/Moment



Magnitude of magnetic torque on a current loop $\tau = IBA\sin\phi$

Current

Magnetic-field magnitude

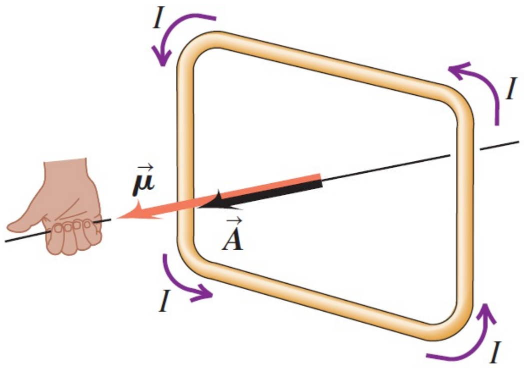
Angle between normal to loop plane and field direction

Area of loop

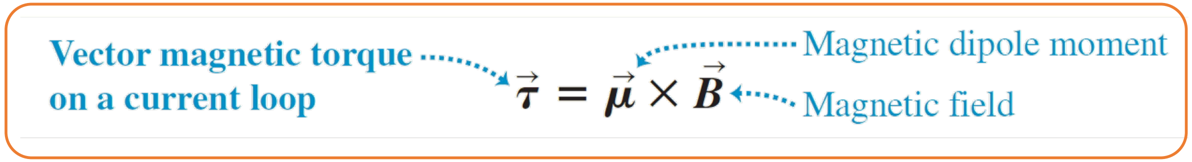
Magnetic Dipole: $\mu = IA$

$$\underline{\tau = \mu B\ sin\phi}$$

Vector Magnetic Dipole: $\vec{\mu} = I\vec{A}$



The direction of $\vec{\mu}$ follows a right-hand rule.



Vector magnetic torque on a current loop $\vec{\tau} = \vec{\mu} \times \vec{B}$

Magnetic dipole moment

Magnetic field

The direction of $\vec{\tau}$ follows a right-hand rule.

(because of the cross product)

Potential Energy of a Current Loop

As the loop rotates due to the torque, the magnetic field does **work** on the loop.

Potential energy for a magnetic dipole in a magnetic field

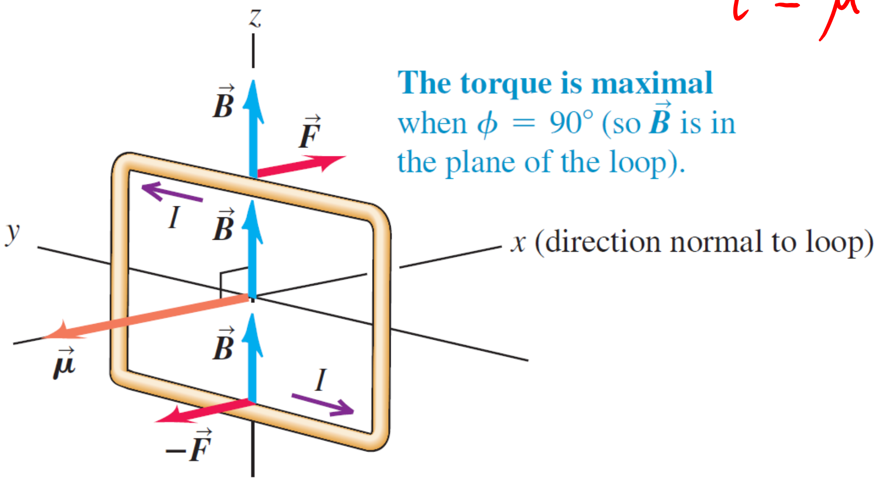
$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi$$

Magnetic dipole moment

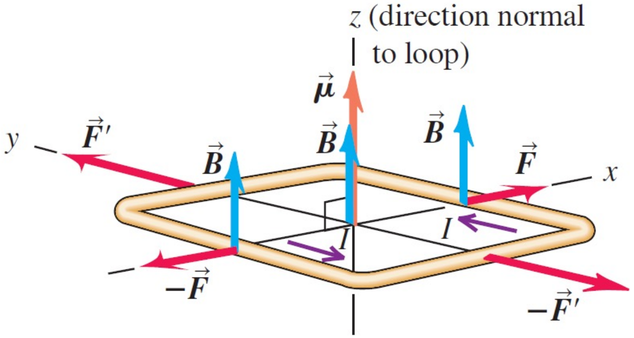
Magnetic field

Angle between $\vec{\mu}$ and $\vec{B}$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin\phi$$



The torque is maximal when $\phi = 90°$ (so $\vec{B}$ is in the plane of the loop).



The torque is zero when $\phi = 0°$ (as shown here) or $\phi = 180°$. In both cases, $\vec{B}$ is perpendicular to the plane of the loop.

The loop is in stable equilibrium when $\phi = 0°$; it is in unstable equilibrium when $\phi = 180°$.

The potential energy is zero.

The potential energy is maximal.

Example

The loop of wire shown in forms a right triangle and carries a current I = 5 A in the direction shown. The loop is in a uniform magnetic field that has magnitude B = 2 T directed into the screen.

Find the net Force exerted by the magnetic field. $\Rightarrow$ 0

Find the net torque on the loop.

$\vec{B} = -2T\,\hat{i}$

$\vec{\tau} = \vec{\mu} \times \vec{B}$

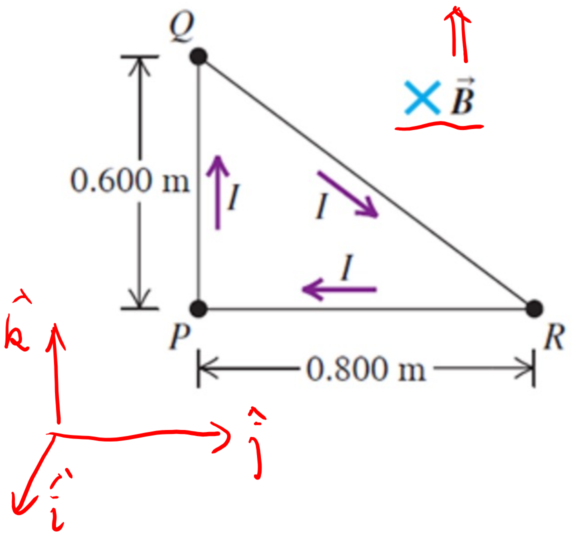
$\vec{\mu} = I\vec{A} = (5A)\,\frac{1}{2}(0.6\,m) \times (0.8\,m)\underline{(-\hat{i})}$

$\Updownarrow$

$\vec{B} \sim (-\hat{i})$

$= (- \;\; - \;\; -)\,\underbrace{(-\hat{i}) \times (-\hat{i})}_{\substack{\shortparallel \\ 0}}$

$= 0$

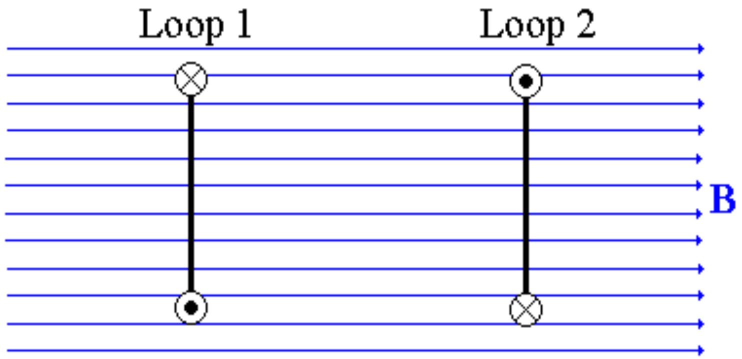


Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed. Which loop has a smaller potential energy?

A. Loop 1

[✓ B. Loop 2]

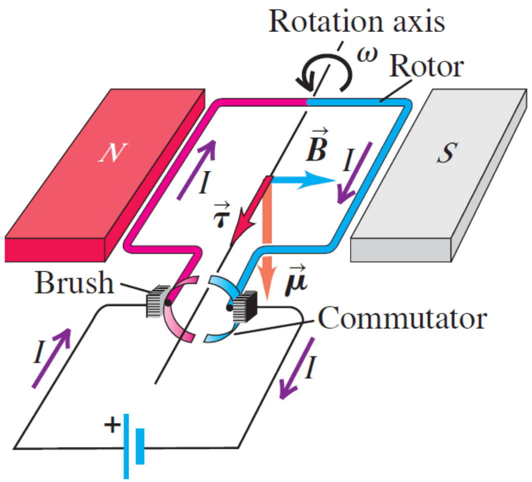
C. Same



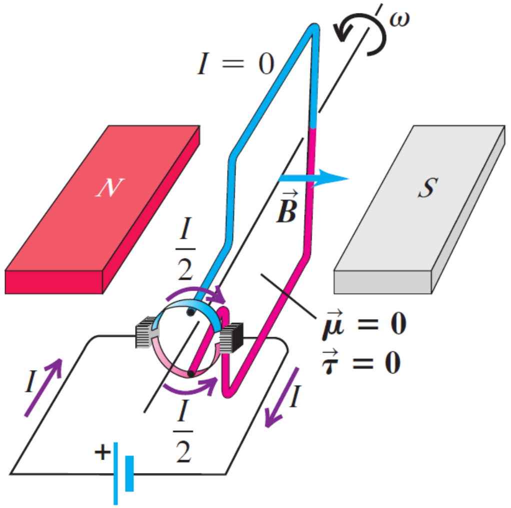
$$U = \textcircled{-}\vec{\mu} \cdot \vec{B}$$

Direct Current Motor

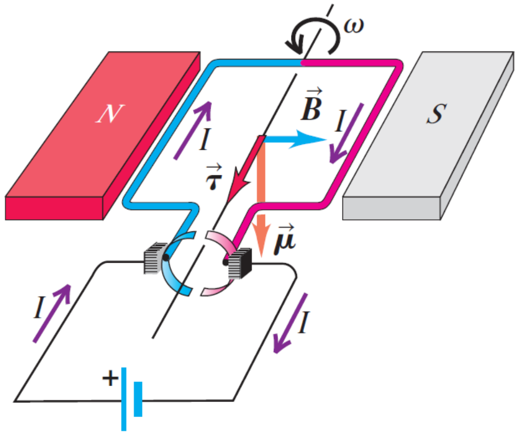
The rotor is a wire loop that is free to rotate about an axis; the rotor ends are attached to the two curved conductors that form the commutator.



- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.



- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.



- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.