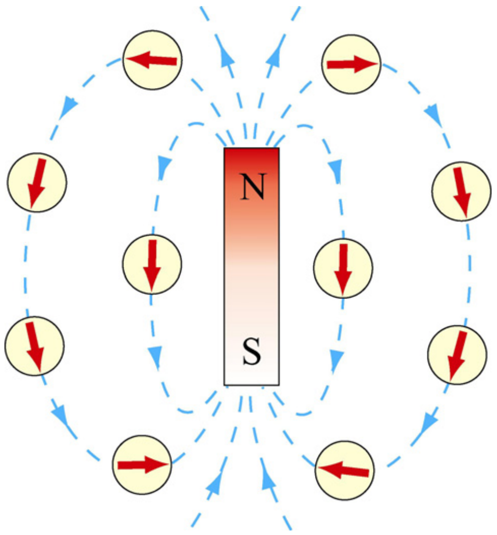
ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 15: Magnetic Field & Magnetic Forces

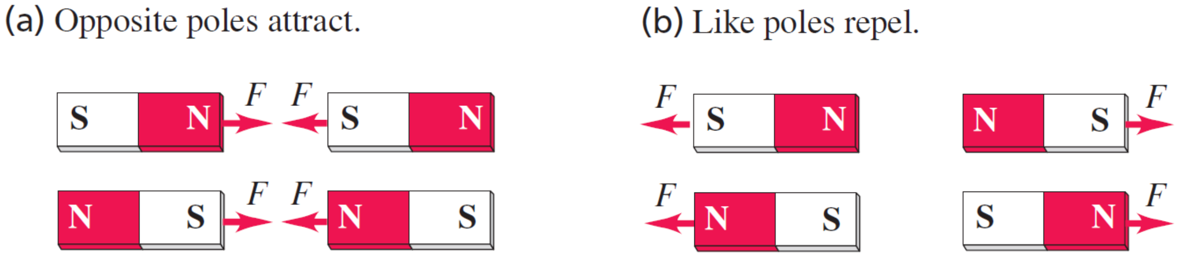
Oct 21, 2024

Magnetic Field

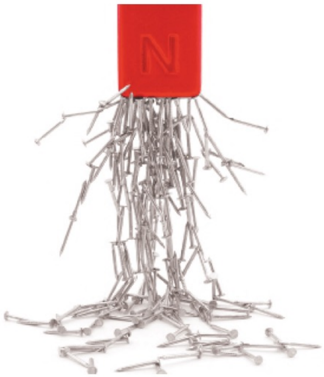


Magnetic field produced by a bar magnet

From north pole to south pole

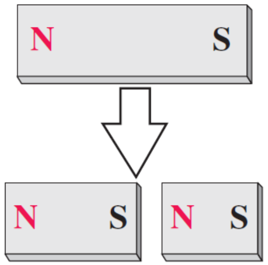


(similar to Coulomb force)



Magnet attracts iron.

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.



Breaking a magnet in two ...

... yields two magnets, not two isolated poles.

There's no magnetic monopole!

Magnetic Field

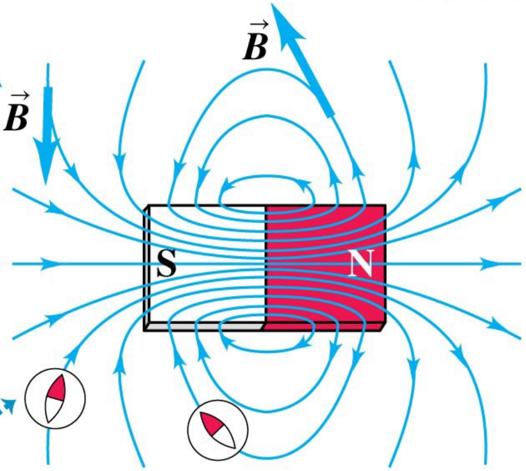
- Magnetic field $\vec{B}$ is a **vector** field and it has both magnitude and direction.
- The SI unit of $\vec{B}$ is **tesla**, 1 tesla$= 1T = 1N/(A \cdot m)$
- Another unit is gauss, 1 gauss$= 10^{-4}T$
- Unlike electric field which begin and end on charges, magnetic field **does not have a beginning or an end** (magnetic charge does not exist).

Magnetic field lines

- The denser, the stronger.
- Magnetic field is tangent to the field line.

At each point, the field line is tangent to the magnetic-field vector $\vec{B}$.

The more densely the field lines are packed, the stronger the field is at that point.



At each point, the field lines point in the same direction a compass would ...

... therefore, magnetic field lines point *away from* N poles and *toward* S poles.

Gauss's Law for Magnetic Field

- Magnetic flux: $\Phi_B = \oiint \vec{B} \cdot d\vec{A}$
- Unit of Φ_B is weber, 1Wb = 1 $T \cdot m^2$

$$\oiint_S \vec{B} \cdot d\vec{A} = 0 \qquad \text{for any closed surface } S$$

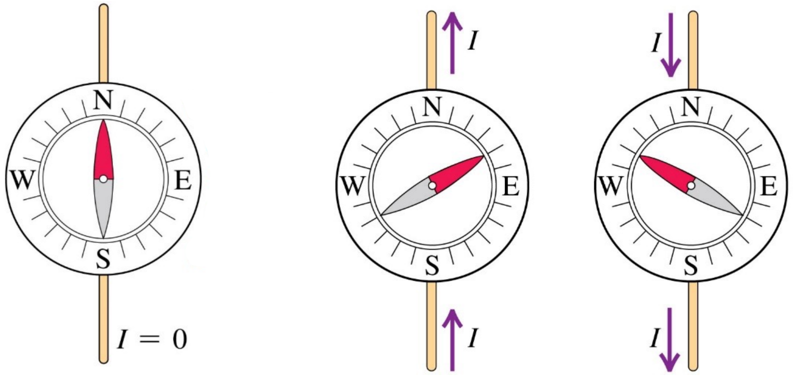
- The net magnetic flux for any closed surface is always zero.
- The 2nd Maxwell equation.
- There is no magnetic monopole.

Electric current and magnets

There is a fundamental relationship between Magnetic Fields and Electric Currents.

↕

Movement of electric charges



Ørsted’s experiment (1820)

1. A moving charge or a current creates a **magnetic field** in the surrounding space (in addition to its *electric* field).
2. The magnetic field exerts a force $\vec{F}$ on any other moving charge or current that is present in the field.

Magnetic Force

Particle's charge

Magnetic force on a moving charged particle → $\vec{\boldsymbol{F}} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$ ← Magnetic field

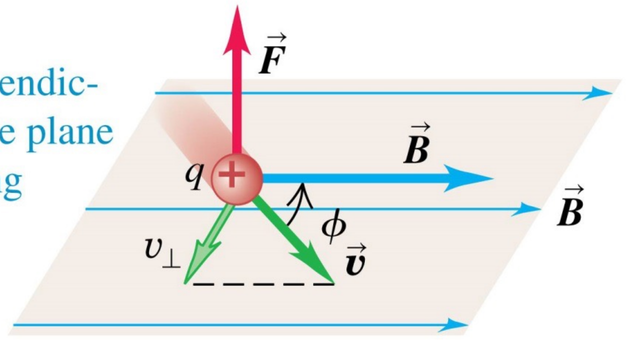
Particle's velocity

Force Magnitude: $F = |q|vB\ sin\phi$

The magnetic force is

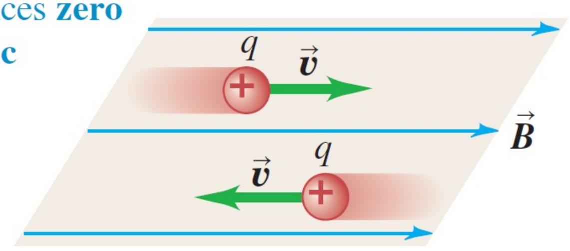
- ○ **Proportional** to the charge q, the speed $\vec{v}$, and $\sin\phi$
- ○ **Perpendicular** to both $\vec{v}$ & $\vec{B}$

$\vec{\boldsymbol{F}}$ is perpendicular to the plane containing $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{B}}$.

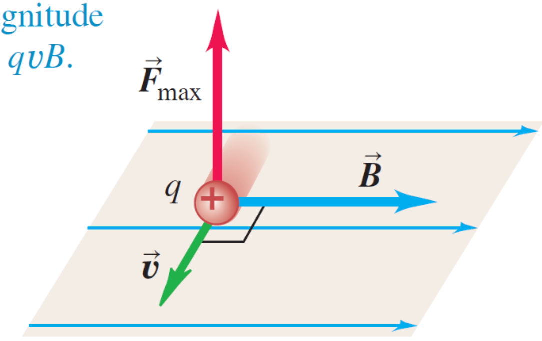


(ϕ is the angle between $\vec{v}$ & $\vec{B}$)

A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**



A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude $F_{\text{max}} = qvB$.



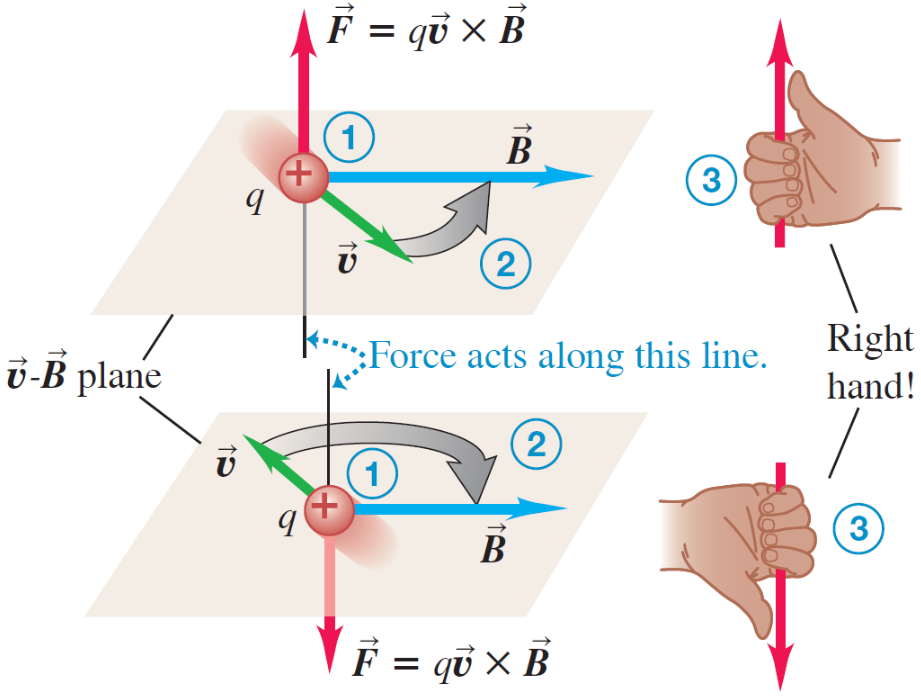
Right-hand Rule

Right-hand rule for the direction of magnetic force on a **positive** charge moving in a magnetic field:

① Place the $\vec{v}$ and $\vec{B}$ vectors tail to tail.

② Imagine turning $\vec{v}$ toward $\vec{B}$ in the $\vec{v}$-$\vec{B}$ plane (through the smaller angle).

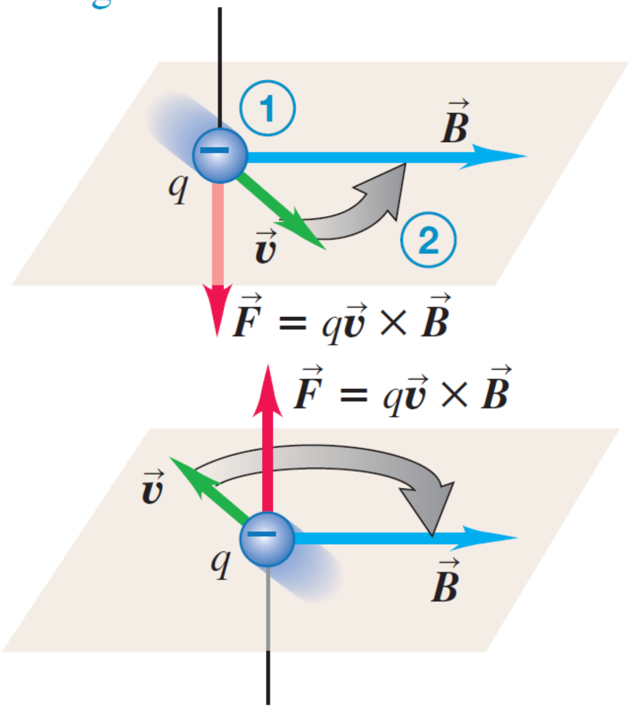
③ The force acts along a line perpendicular to the $\vec{v}$-$\vec{B}$ plane. Curl the fingers of your *right hand* around this line in the same direction you rotated $\vec{v}$. Your thumb now points in the direction the force acts.



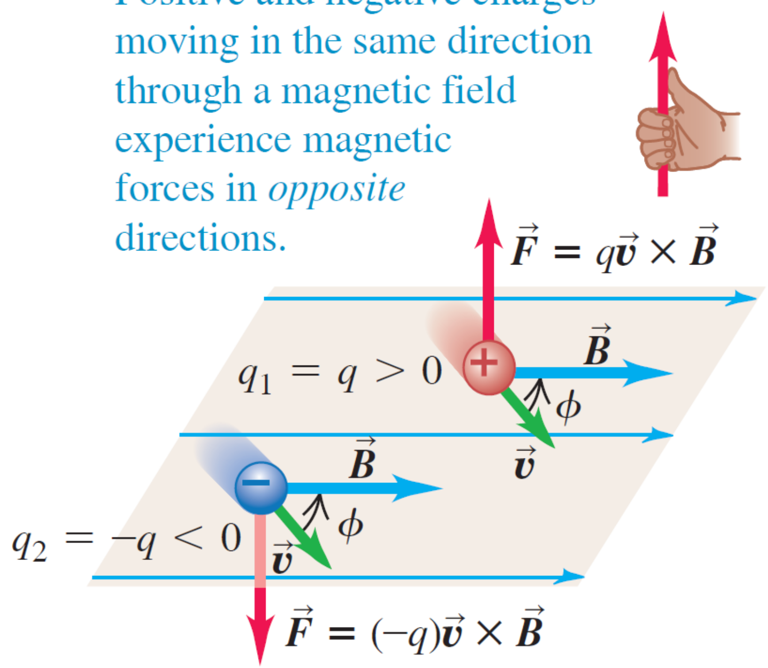
Warning: Use your right hand even if you are left-handed!

Right Hand Rule for Negative Charges

If the charge is negative, the direction of the force is ***opposite*** to that given by the right-hand rule.



Positive and negative charges moving in the same direction through a magnetic field experience magnetic forces in *opposite* directions.



Flipping the sign of charge also flips the direction of magnetic force.

Vector Product

The vector (or cross) product between two vectors $\vec{A}$ & $\vec{B}$ is also a vector $\vec{C} = \vec{A}\times\vec{B}$, s.t.

- ○ $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ (right-hand rule)
- ○ $|\vec{C}| = \underline{AB\sin\phi}$, where ϕ is the angle between $\vec{A}$ and $\vec{B}$.

Vector product of Cartesian coordinate components

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}},\quad \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}},\quad \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

$$\hat{\mathbf{j}}\times\hat{\mathbf{i}} = -\hat{\mathbf{k}},\quad \hat{\mathbf{k}}\times\hat{\mathbf{j}} = -\hat{\mathbf{i}},\quad \hat{\mathbf{i}}\times\hat{\mathbf{k}} = -\hat{\mathbf{j}}$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0$$

Example: Calculate $\vec{A}\times\vec{B}$ with $\vec{A} = (1,2,0)$ and $\vec{B} = (0,1,3)$

$$\vec{A} = \hat{i} + 2\hat{j}\ ,\quad \vec{B} = \hat{j} + 3\hat{k}$$

$$\vec{A}\times\vec{B} = (\hat{i} + 2\hat{j}) \times (\hat{j} + 3\hat{k})$$

$$= \hat{i}\times\hat{j} + \hat{i}\times 3\hat{k} + 2\hat{j}\times\hat{j} + 2\hat{j}\times 3\hat{k}$$

$$= \hat{k} - 3\hat{j} + 0 + 6\hat{i}$$

$$= 6\hat{i} - 3\hat{j} + \hat{k} = (6,-3,1)$$

Example

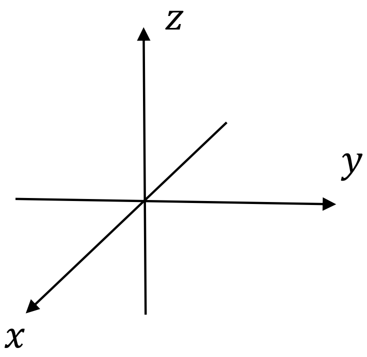
A particle with a charge of $q = 1\times10^{-8}C$ is moving with an instantaneous velocity $\vec{v} = 4\times10^4\,\frac{m}{s}\,\hat{\imath} - 3\times10^4\,\frac{m}{s}\,\hat{\jmath}$. What is the force exerted on the particle by a magnetic field $\vec{B} = 1.5T\hat{\imath}$?

$$\vec{F} = q\vec{v}\times\vec{B} = 1\times10^{-8}C\left[(4i-3j)\times10^{4}\frac{m}{s}\times1.5T\,i\right]$$

$$= 1\times10^{-8}C\times10^{4}\ \left(\underbrace{4i\times1.5i}_{\overset{\shortparallel}{0}} - \underbrace{3j\times1.5i}_{-k\,4.5}\right)\frac{m}{s}\,T$$

$$= 1\times10^{-4}\,(+4.5\hat{k})\,\frac{m}{s}\,T\cdot C$$

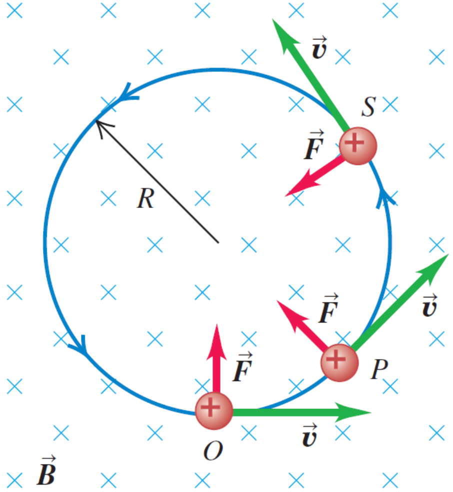
$$= \underline{4.5\times10^{-4}\,N\,\hat{k}}$$



Motion of a Charged Particle under B

When a charged particle moves in a magnetic field it experiences a force that is perpendicular to the velocity.

1) The magnetic force does **NO** work on the particle.
2) The acceleration is **perpendicular** to $\vec{v}$.
3) The acceleration only changes the **direction** of $\vec{v}$, but **NOT** its magnitude.
4) The magnetic force only provides a **centripetal acceleration**.



$$F = \boxed{|q|vB} = m\boxed{\frac{v^2}{R}}$$

magnetic force

centripetal acceleration

Radius of a circular orbit in a magnetic field

$$R = \frac{mv}{|q|B}$$

Particle's mass

Particle's speed

Magnetic-field magnitude

Particle's charge

Motion of a Charged Particle under B

Radius of a circular orbit in a magnetic field

$$R = \frac{mv}{|q|B}$$

Particle's mass

Particle's speed

Magnetic-field magnitude

Particle's charge

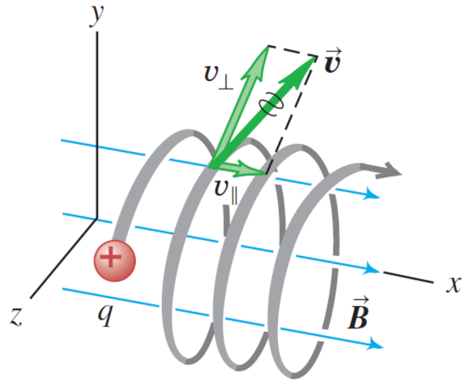
This is actually the velocity component $v_\perp$ **perpendicular** to the magnetic field

Angular speed $$\omega = \frac{v}{R} = \frac{|q|B}{m}$$

Cyclotron frequency $$f = \frac{\omega}{2\pi} = \frac{|q|B}{2\pi m}$$

(# of revolution per unit time)

This particle's motion has components both parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the magnetic field, so it moves in a helical path.

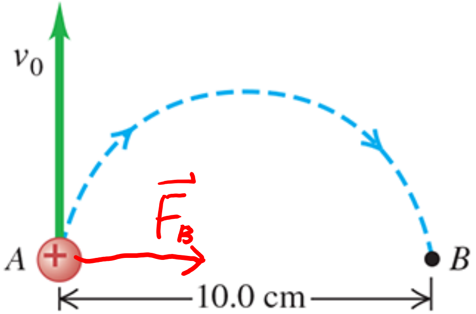


Example

A proton at point A has a speed vo of $1.6\times10^{-6} m/s$. (m_p , q_p are the mass & charge for protons)

Find the magnetic field that will cause the proton to follow the semicircular path from A to B.

Find the time required for the proton to move from A to B.



a) RHR ⇒ $\vec{B}$ is out-of-page

$$R = \frac{10cm}{2} = 5\times10^{-2} m = \frac{m_p V_0}{q_p B} = \frac{m_p}{q_p}\frac{1.6\times10^{-6} m/s}{B}$$

$$\Rightarrow \quad B = \frac{m_p}{q_p}\frac{1.6\times10^{-6} m/s}{5\times10^{-2} m} = \cdots$$

b) Arc length $= \pi\cdot R = t\cdot V_0 \Rightarrow t = \frac{\pi R}{V_0} = \pi\frac{5\times10^{-2} m}{1.6\times10^{-6} m/s} \simeq \frac{50\pi}{16}\times10^{4} s$