ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 14: Direct-Current Circuits (R-C)

Oct 16, 2024

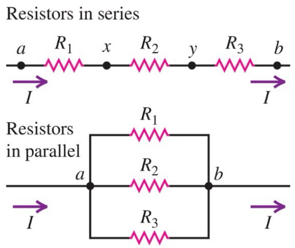
Chapter summary

(26.1)

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$
(resistors in series)

(26.2)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$
(resistors in parallel)



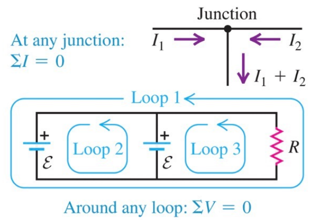
(26.5)

$$\sum I = 0$$
(junction rule)

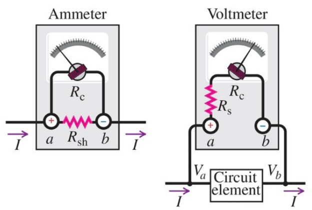
Kirchhoff's rule

(26.6)

$$\sum V = 0$$
(loop rule)



Electrical measuring instruments: In a d'Arsonval galvanometer, the deflection is proportional to the current in the coil. For a larger current range, a shunt resistor is added, so some of the current bypasses the meter coil. Such an instrument is called an ammeter. If the coil and any additional series resistance included obey Ohm's law, the meter can also be calibrated to read potential difference or voltage. The instrument is then called a voltmeter. A good ammeter has very low resistance; a good voltmeter has very high resistance. (See Examples 26.8, 26.9, 26.10 and 26.11.)



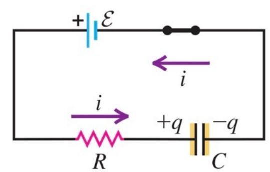
Capacitor charging:

(26.12)

$$\begin{aligned} q &= C\epsilon(1 - e^{-t/RC}) \\ &= Q_f(1 - e^{-t/RC}) \end{aligned}$$

(26.13)

$$i = \frac{dq}{dt} = \frac{\epsilon}{R} e^{-t/RC} = I_0 e^{-t/RC}$$



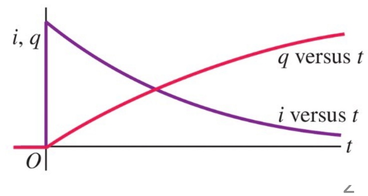
Capacitor discharging:

(26.16)

$$q = Q_0 e^{-t/RC}$$

(26.17)

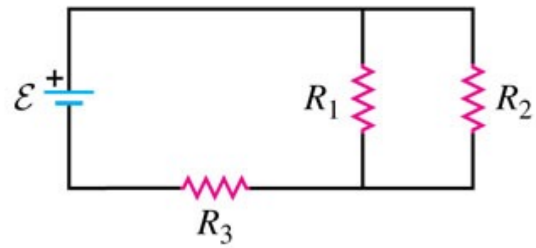
$$i = \frac{dq}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$



Example 3

$\varepsilon = 8.0\ V, R_1 = 3.0\Omega, R_3 = 2.0\Omega,$ & R_2 can vary between 3.0Ω and 6.0Ω.

1) For what value of R_2 is the power dissipated by R_1 the greatest?
2) What is that power?



According to $P = \frac{V^2}{R}$, to make $P_1 \uparrow \Rightarrow V_1 \uparrow$

This is why we want $R_2 \uparrow$

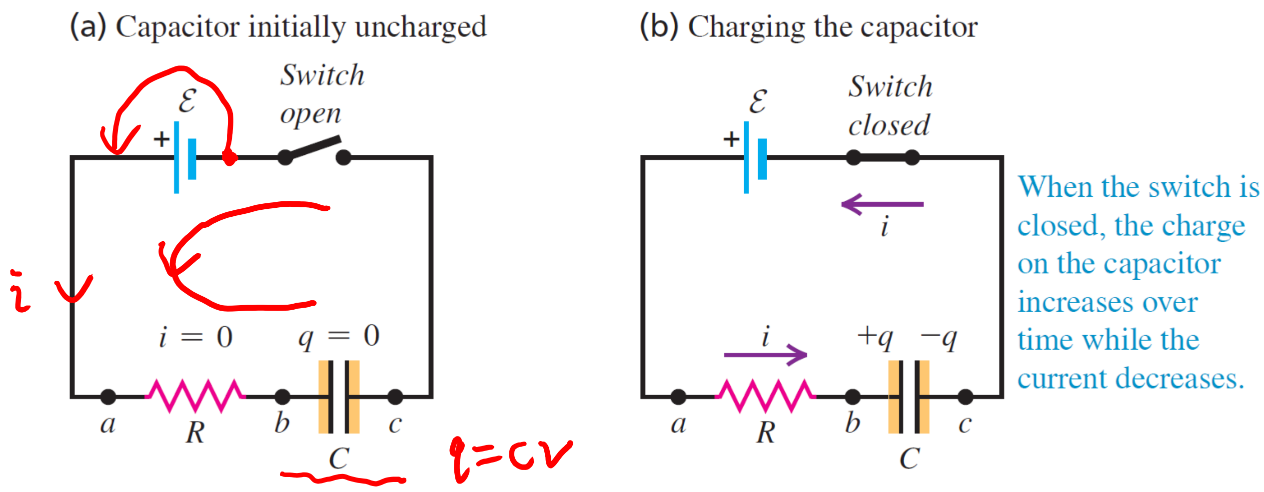
1) $(R_2)_{max} = 6\Omega$

2) $R_{12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{3} + \frac{1}{6}} = 2\Omega$, $R_{123} = R_3 + R_{12} = 4\Omega$

$\Rightarrow I = \frac{\varepsilon}{R_{123}} = \frac{8V}{4\Omega} = 2A \Rightarrow V_3 = I R_3 = 4V$

$\Rightarrow V_1 = \varepsilon - V_3 = 8V - 4V = 4V \Rightarrow P = \frac{V_1^2}{R_1} = \frac{(4V)^2}{3\Omega} = \frac{16}{3} W.$

Charging a Capacitor



$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C}$$

Kirchhoff's loop rule: $\mathcal{E} - iR - \dfrac{q}{C} = 0$ ➡ $i = \dfrac{\mathcal{E}}{R} - \dfrac{q}{RC}$ ➡ $\dfrac{dq}{dt} = \dfrac{\mathcal{E}}{R} - \dfrac{q}{RC} = -\dfrac{1}{RC}(q - C\mathcal{E})$

i, q function of t.

See textbook

R-C circuit, charging capacitor:

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC})$$

Capacitor charge; Battery emf; Capacitance; Time since switch closed; Resistance; Final capacitor charge $= C\mathcal{E}$

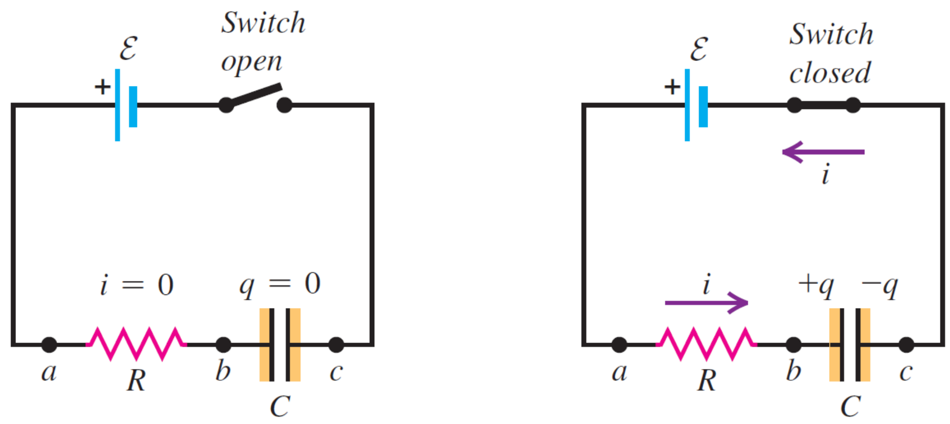
$\begin{cases} t=0,\ q=0 \\ t=\infty,\ q=\mathcal{E}C = Q_f \end{cases}$

$\begin{cases} e^{-\infty}=0 \\ e^{0}=1 \end{cases}$

Charging a Capacitor

(a) Capacitor initially uncharged

(b) Charging the capacitor



When the switch is closed, the charge on the capacitor increases over time while the current decreases.

$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C}$$

Kirchhoff's loop rule: $\mathcal{E} - iR - \frac{q}{C} = 0 \Rightarrow i = \frac{\mathcal{E}}{R} - \frac{q}{RC} \Rightarrow \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E})$

$$\frac{d(q - C\epsilon)}{dt} = -\frac{1}{RC}(q - C\epsilon)$$

$$\frac{d(q - C\epsilon)}{q - C\epsilon} = -\frac{dt}{RC}$$

$$\ln(q - C\epsilon) = -\frac{t}{RC} + const'$$

$$q - C\epsilon = const * e^{-\frac{t}{RC}}$$

With initial condition t=0, q=0, $const$ is fixed to $-C\epsilon$.

$$q = C\epsilon(1 - e^{-t/RC})$$

***R-C* circuit, charging capacitor:**

Capacitor charge · Capacitance · Final capacitor charge $= C\mathcal{E}$

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC})$$

Battery emf · Time since switch closed · Resistance

Charging a Capacitor

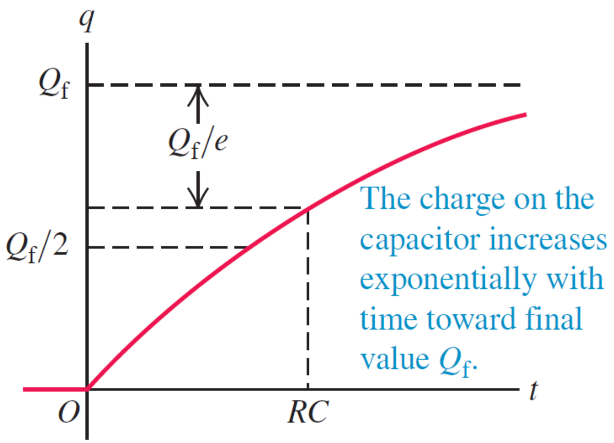
***R-C* circuit, charging capacitor:**

Capacitor charge · Capacitance · Final capacitor charge $= C\mathcal{E}$

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

Battery emf · Time since switch closed · Resistance

(b) Graph of capacitor charge versus time for a charging capacitor



The charging speed decays as a function of time…

Like a really really hungry guy walking into a buffet…

Initially, he eats as fast as he could because he is so hungry.

He gradually slow down eating as he is not that hungry, until he is completely full…

Charging a Capacitor

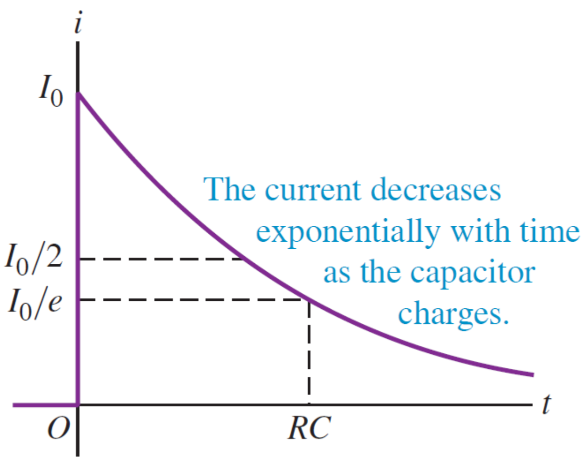
***R-C* circuit, charging capacitor:**

Current　Battery emf　Time since switch closed

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

Initial current $= \mathcal{E}/R$

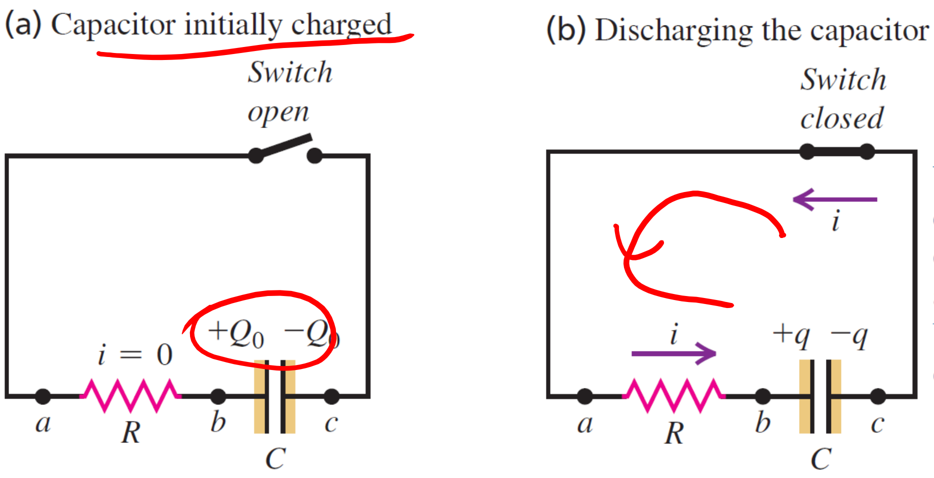
Rate of change of capacitor charge　Resistance　Capacitance



Time constant: $\tau = RC$

τ is a measure of how quickly the capacitor charges.

Discharging a Capacitor



When the switch is closed, the charge on the capacitor and the current both decrease over time.

time constant $\tau = RC$

$$-iR - \frac{q}{C} = 0$$

$$i = \frac{dq}{dt}$$

$$\Rightarrow \frac{dq}{dt} = -\frac{q}{RC}$$

R-C circuit, discharging capacitor:

$$q = Q_0 e^{-t/RC}$$

Capacitor charge; Initial capacitor charge; Capacitance; Resistance; Time since switch closed

R-C circuit, discharging capacitor:

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

Current; Rate of change of capacitor charge; Initial capacitor charge; Capacitance; Resistance; Time since switch closed; Initial current $= -Q_0/RC$

$\Rightarrow$ current flowing clockwise

The negative sign of the current i indicates it's flowing opposite to the purple arrows

Notes

Basic principle: Capacitor **resists rapid changes** in Q & V

Charging

- Initially, the capacitor behaves like a **wire** (V = 0, since Q = 0).
- As current starts to flow, charge builds up on the capacitor.

1) it then becomes more difficult to add more charge
2) the current decreases

- After a long time, the capacitor behaves like **an open switch**. $\Leftrightarrow$ R_∞

Discharging

- Initially, the capacitor behaves like a **battery**.
- After a long time, the capacitor behaves like a **wire**.

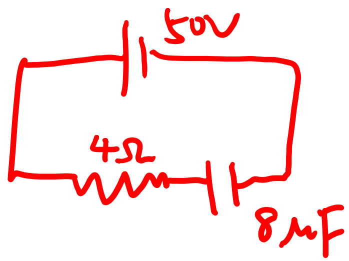
Example

A 8.00 μF capacitor that is initially uncharged is connected in series with a 4.00 Ω resistor and an emf source with E = 50.0 V and negligible internal resistance.

Q: At the instant when the resistor is dissipating electrical energy at a rate of 400 W, how much energy has been stored in the capacitor?

$\Downarrow$

$P_R = 400\,W$



$P_R = I^2 R = \frac{V^2}{R} \Rightarrow 400W = \frac{V_R^2}{4\Omega}$

$\Rightarrow V_R = \sqrt{400 \times 4}\ V = \sqrt{1600}\ V = 40V$

$V_C = \mathcal{E} - V_R = 50V - 40V = 10V$

$U_C = \frac{1}{2} C \cdot V_C^2 = \frac{1}{2} \times 8 \times 10^{-6} F \times (10V)^2 = 4 \times 10^{-4} J$

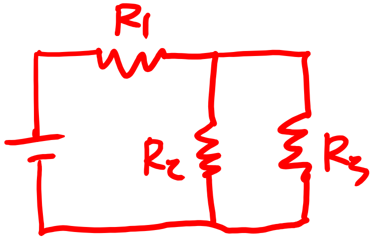
Hint: Energy stored in a capacitor is $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

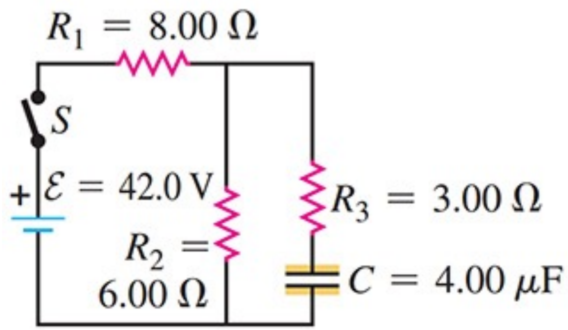
Example

The capacitor in (Figure 1) is initially uncharged. The switch is closed at t=0.

a) Immediately after the switch is closed, what is the current through the resistor R1, R2, R3?

b) What is the final charge on the capacitor?





a) Capacitor behaves like a wire

$$R_{23} = \frac{1}{\frac{1}{R_2}+\frac{1}{R_3}} = 2\Omega$$

$$R_{123} = R_1 + R_{23} = 10\Omega \Rightarrow I = I_1 = \frac{\varepsilon}{R_{123}} = \frac{42V}{10\Omega} = 4.2A$$

$$V_1 = I_1 R_1 = 8\Omega \times 4.2A = 33.6V \Rightarrow V_2 = V_3 = \varepsilon - V_1 = 42V - 33.6V = 8.4V$$

$$I_2 = \frac{V_2}{R_2} = \frac{8.4}{6} A, \quad I_3 = \frac{8.4V}{R_3} = 2.8A$$

b) Find V_f, Capacitor $\Rightarrow$ open switch $\Rightarrow$

$Q = V_f C$ $\Rightarrow$ $= 18V \times 4\mu F$ $= \ldots$

$$I = \frac{\varepsilon}{8\Omega + 6\Omega} = \frac{42V}{14\Omega} = 3A \Rightarrow V_2 = IR_2 = 3A \times 6\Omega = 18V = V_f$$

Example

You connect a battery, resistor, and capacitor as in (Figure 1), where E = 46.0 V, C = 5.00 μF, and R = 120 Ω. The switch S is closed at t = 0. When the voltage across the capacitor is 8.00 V

a) What is the magnitude of the current in the circuit?
b) What is t?
c) At what rate is energy being stored in the capacitor?