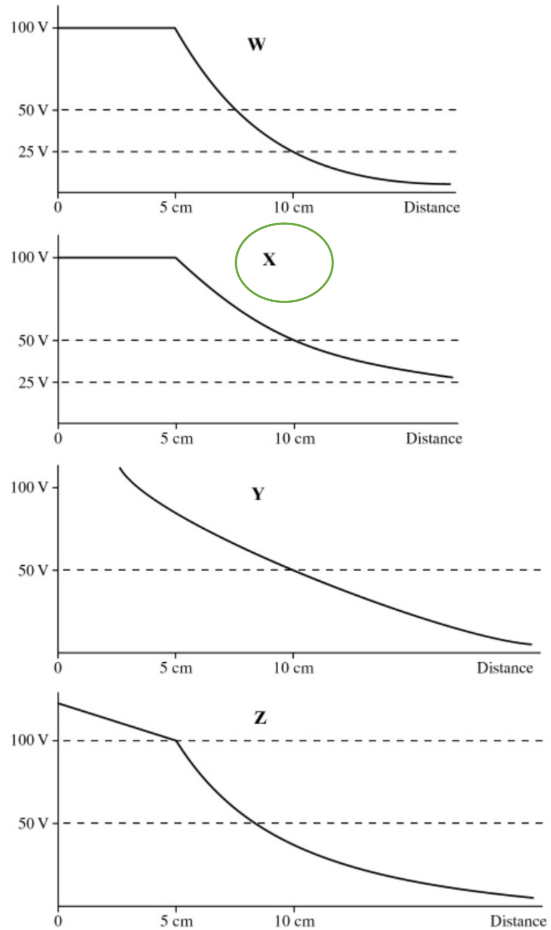
Midterm #1

1) Spheres of charge: A metallic sphere of radius 5 cm is charged such that the potential of its surface is 100 V (relative to infinity). Which of the following plots correctly shows the potential as a function of distance from the center of the sphere?



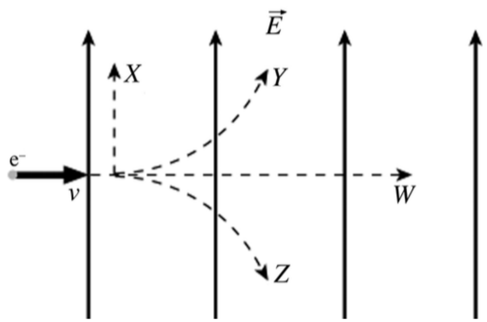
2) Dielectrics: A charged capacitor stores energy U. Without connecting this capacitor to anything, dielectric having dielectric constant K is now inserted between the plates of the capacitor, completely filling the space between them. How much energy does the capacitor now store? 2) E

A) $\frac{U}{2K}$ B) KU C) $2KU$ D) U E) $\frac{U}{K}$

3) Coulomb's law: Two identical small charged spheres are a certain distance apart, and each one initially experiences an electrostatic force of magnitude F due to the other. With time, charge gradually leaks off of both spheres. When each of the spheres has lost half its initial charge, the magnitude of the electrostatic force will be 3) B

A) 1/8 F. B) 1/4 F. C) 1/2 F. D) 1/16 F.

4) Motion of a charged particle: An electron is initially moving to the right when it enters a uniform electric field directed upwards. Which trajectory shown below will the electron follow? 4) D



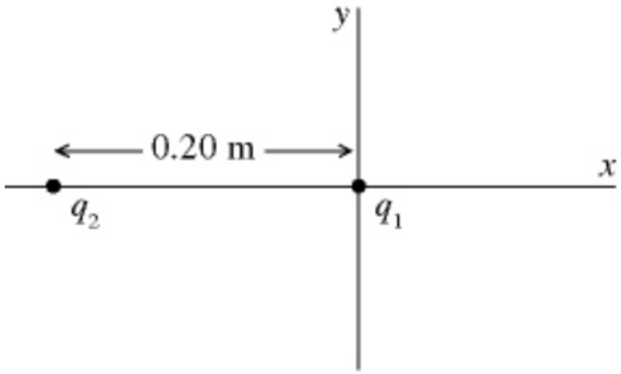
A) trajectory X B) trajectory W C) trajectory Y D) trajectory Z

5) Ohm's law: When a potential difference of 10 V is placed across a certain solid cylindrical resistor, the current through it is 2 A. If the diameter of this resistor is now tripled, the current will be 5) C

A) 2 A. B) 3 A. C) 18 A. D) 2/3 A. E) 2/9 A.

Midterm #1

6) Coulomb's law: In the figure, charge $q_1 = 3.1 \times 10^{-6}$ C is placed at the origin and charge $q_2 = -8.7 \times 10^{-6}$ C is placed on the x-axis, at $x = -0.20$ m. Where along the x-axis can a third charge $Q = -8.3$ μC be placed such that the resultant force on this third charge is zero? (hint: right side of x axis)



$$q * \frac{q_2}{(x+0.2)^2} + q * \frac{q_1}{x^2} = 0$$

$$q_2 * x^2 = -q_1 * (x+0.2)^2$$

$$2.806 * x^2 = (x+0.2)^2$$

$$x = 0.5\ m$$

ELECTRICITY AND MAGNETISM (PHYS 231)

Lecture 13: Direct-Current Circuits

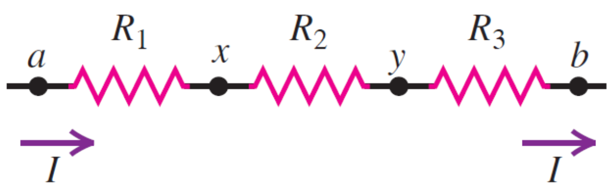
Oct 14, 2024

Resistors in Series & Parallel

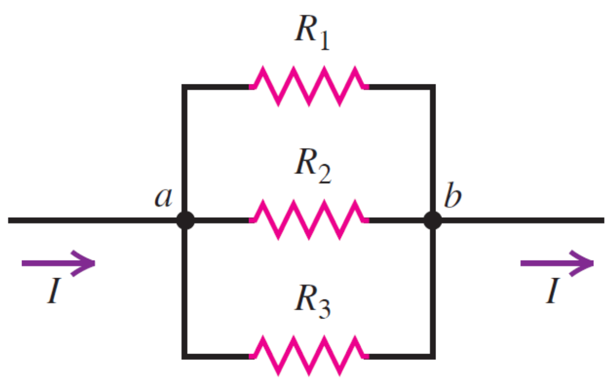
Direct-current (dc) circuits: The direction of the current does not change with time.

Alternating current (ac) circuits: The direction of the current reverses periodically.

Different ways of combining resistors

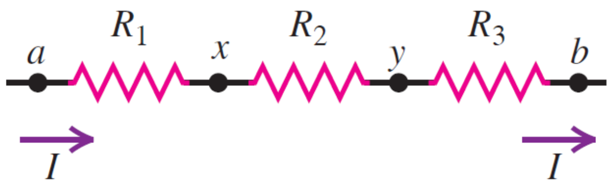


Resistors in series



Resistors in parallel

Resistors in Series



$V_1 = I_1 R_1, \qquad V_2 = I_2 R_2, \qquad V_3 = I_3 R_3$

Resistors in series share the **SAME current** I.

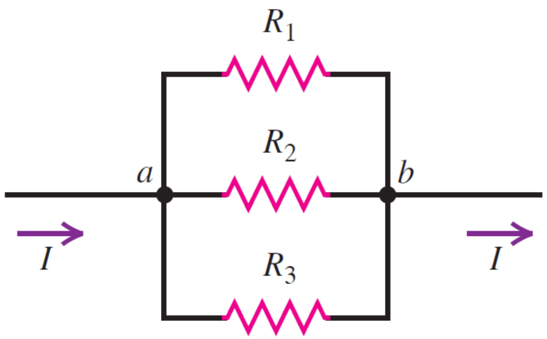
Find R_{eq}, the equivalent resistance for the resistors in series.

Since $I_1 = I_2 = I_3 = I$ ➡ $V = IR_{eq} = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum_i R_i$$

Equivalent resistance for the resistors in series

Resistors in Parallel



Resistors in parallel share the **SAME voltage** V.

$$V_1 = I_1 R_1, \qquad V_2 = I_2 R_2, \qquad V_3 = I_3 R_3$$

Find R_{eq}, the equivalent resistance for the resistors in parallel.

Since $V_1 = V_2 = V_3 = V$ ➡

$$R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2 + I_3} = \frac{V}{\frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum_i \frac{1}{R_i}$$

Equivalent resistance for the resistors in parallel

Resistors v.s. Capacitors

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum_i R_i$$

(series)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum_i \frac{1}{R_i}$$

(parallel)

The relations are reversed.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum_i \frac{1}{C_i}$$

(series)

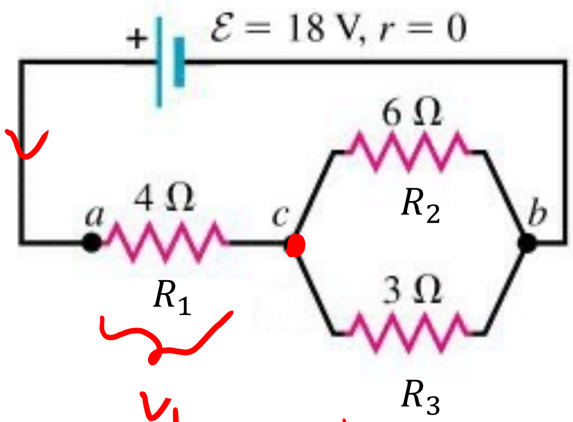
$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum_i C_i$$

(parallel)

Example 1

What is the equivalent resistance?

What is the current through each resistor?



I, V_1, V_{23}

$$R_{23} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{6} + \frac{1}{3}} = \frac{1}{\frac{1}{2}} = 2\Omega$$

$$R_{eq} = R_{123} = R_1 + R_{23} = 4\Omega + 2\Omega = 6\Omega$$

$$I = I_{R_1} = \frac{\mathcal{E}}{R_{eq}} = \frac{18V}{6\Omega} = 3A$$

$$V_1 = I R_1 = 3A \cdot 4\Omega = 12V$$

$$V_{23} = \mathcal{E} - V_1 = 18V - 12V = 6V \Rightarrow$$

$$I_2 = \frac{V_{23}}{R_2} = \frac{6V}{6\Omega} = 1A$$

$$I_3 = \frac{V_{23}}{R_3} = \frac{6V}{3\Omega} = 2A$$

Check $I_2 + I_3 = I$

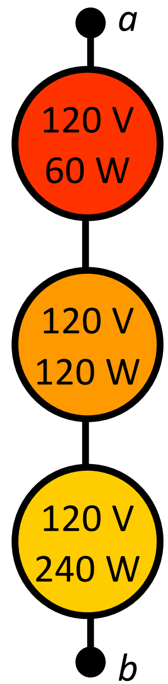
A 120-V, 60-W incandescent light bulb; a 120-V, 120-W incandescent light bulb; and a 120-V, 240-W incandescent light bulb are connected in series as shown. The voltage between points *a* and *b* is 120 V. Through which bulb is there the greatest voltage drop?

A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

C. the 120-V, 240-W light bulb

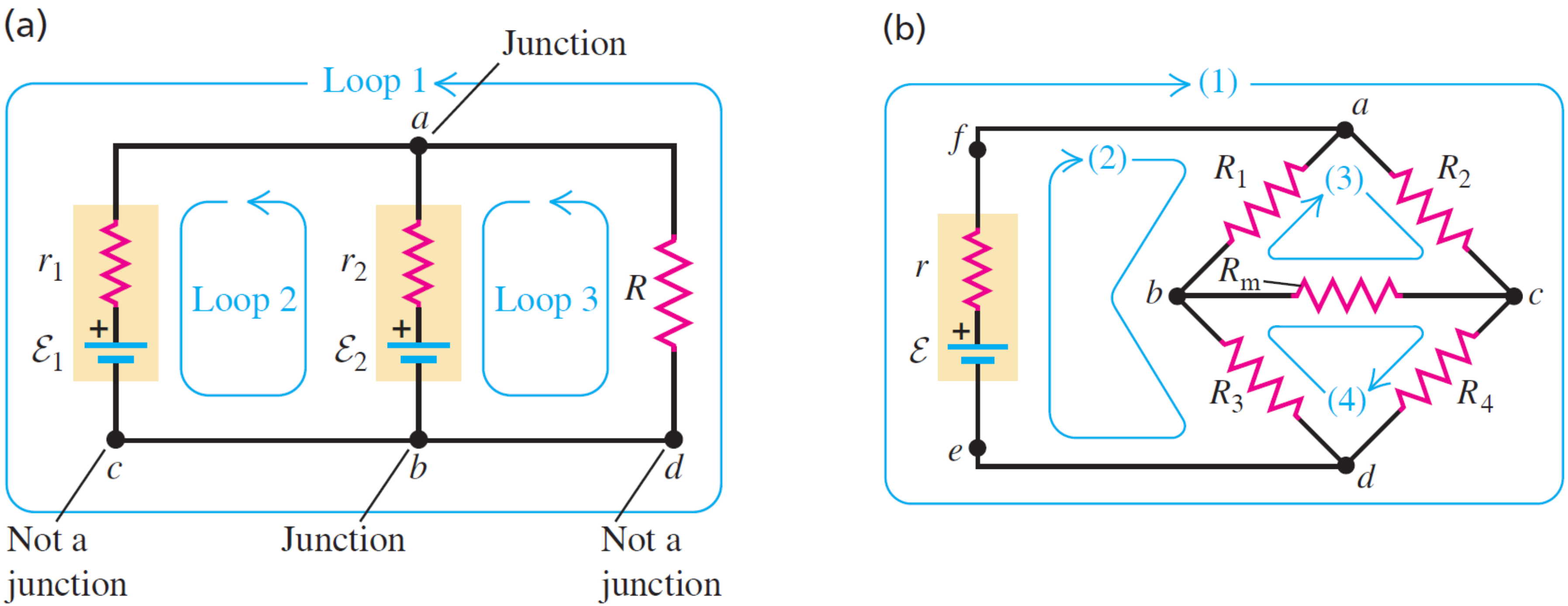
D. The voltage drops across all light bulbs are the same.



$$P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{(120V)^2}{P}$$

Beyond Series & Parallel



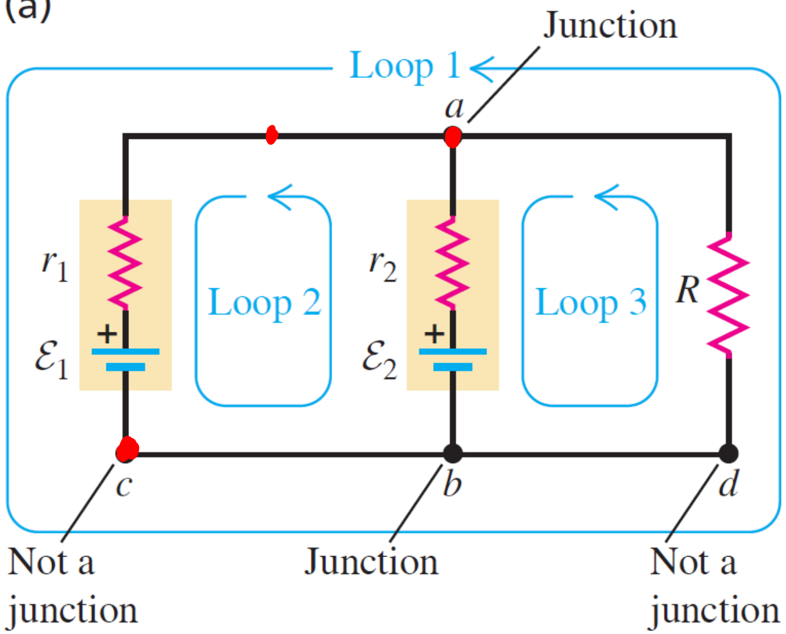
Resistors that are neither in parallel nor in series…

Kirchhoff's Rule: The Ultimate Solution for Circuits

Junction: A point in a circuit where three or more conductors meet.

Loop: Any closed conducting path.

(a)



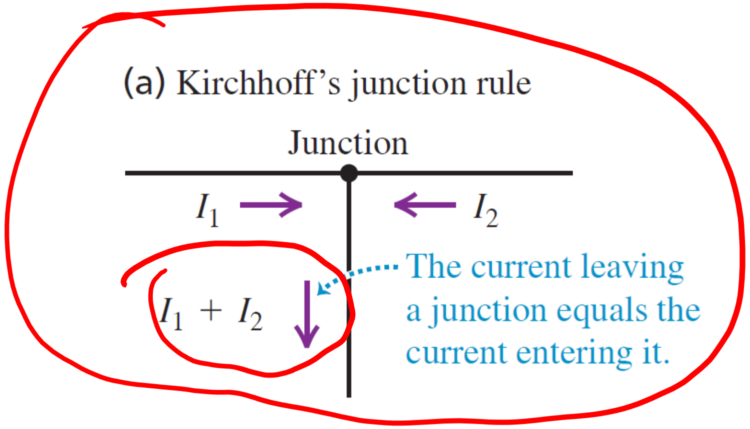
Junction Rule

Kirchhoff's junction rule
(valid at any junction):

The sum of the currents into any junction ...

$$\sum I = 0$$

... equals zero.



Junction rule: $I_1 + I_2 - (I_1 + I_2) = 0$

The tricky part for applying the junction rule lies in the **sign** of the current.

The above sum rule only applies to currents that flow **into** the junction.

If there is a current I that is leaving the junction, it is equivalent to a $-I$ current flowing into the junction.

Loop Rule

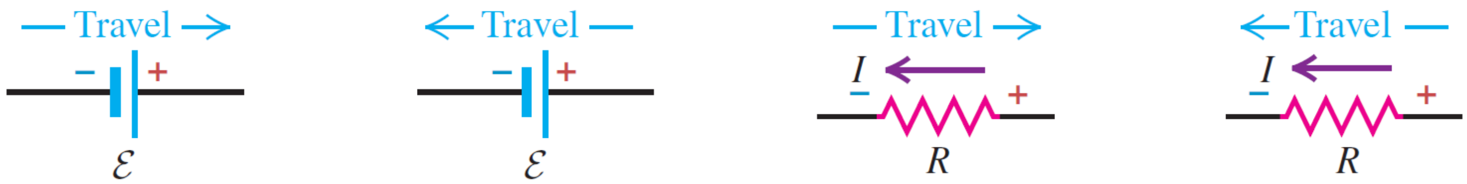
Kirchhoff's loop rule (valid for any closed loop): $\sum V = 0$

The sum of the potential differences around any loop ... equals zero.

(a) Sign conventions for emfs

(b) Sign conventions for resistors

$+\mathcal{E}$: Travel direction from – to +:	$-\mathcal{E}$: Travel direction from + to –:	$+IR$: Travel *opposite to* current direction:	$-IR$: Travel *in* current direction:



Be careful about the sign!

Read This Carefully (Page 851)

PROBLEM-SOLVING STRATEGY 26.2 Kirchhoff's Rules

IDENTIFY *the relevant concepts:* Kirchhoff's rules are useful for analyzing any electric circuit.

SET UP *the problem* using the following steps:

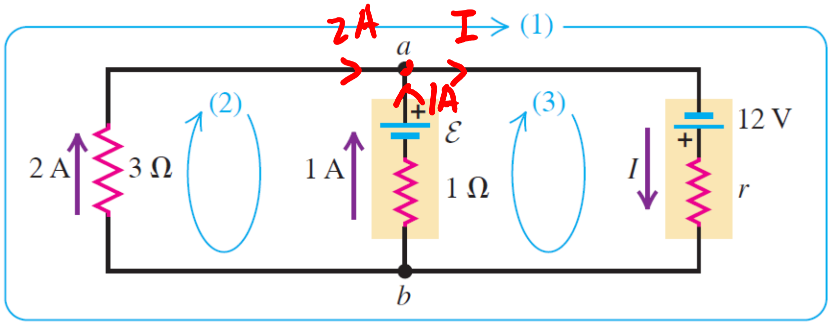
1. Draw a circuit diagram, leaving room to label all quantities, known and unknown. Indicate an assumed direction for each unknown current and emf. (Kirchhoff's rules will yield the magnitudes *and directions* of unknown currents and emfs. If the actual direction of a quantity is opposite to your assumption, the resulting quantity will have a negative sign.)
2. As you label currents, it's helpful to use Kirchhoff's junction rule, as in **Fig. 26.9**, so as to express the currents in terms of as few quantities as possible.
3. Identify the target variables.

EXECUTE *the solution* as follows:

1. Choose any loop in the network and choose a direction (clockwise or counterclockwise) to travel around the loop as you apply Kirchhoff's loop rule. The direction need not be the same as any assumed current direction.
2. Travel around the loop in the chosen direction, adding potential differences algebraically as you cross them. Use the sign conventions of Fig. 26.8.
3. Equate the sum obtained in step 2 to zero in accordance with the loop rule.
4. If you need more independent equations, choose another loop and repeat steps 1–3; continue until you have as many independent equations as unknowns or until every circuit element has been included in at least one loop.
5. Solve the equations simultaneously to determine the unknowns.
6. You can use the loop-rule bookkeeping system to find the potential V_{ab} of any point a with respect to any other point b. Start at b and add the potential changes you encounter in going from b to a; use the same sign rules as in step 2. The algebraic sum of these changes is $V_{ab} = V_a - V_b$.

EVALUATE *your answer:* Check all the steps in your algebra. Apply steps 1 and 2 to a loop you have not yet considered; if the sum of potential drops isn't zero, you've made an error somewhere.

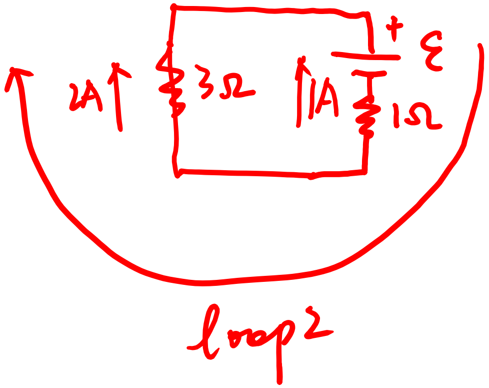
Example 26.4



Find $\varepsilon, r, \& I$.

direction (flowing out)

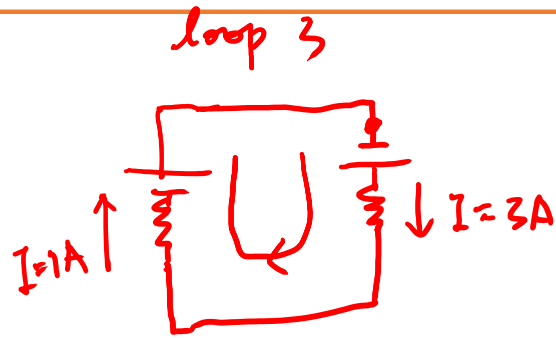
Junction a: $2A + 1A - I = 0 \Rightarrow I = 3A$



$-\varepsilon + 1A \cdot 1\Omega - 2A \cdot 3\Omega = 0$

$\varepsilon = 1V - 6V = -5V$

get changed



$12V - 3A \cdot r - 1A \cdot 1\Omega + (-5V) = 0$

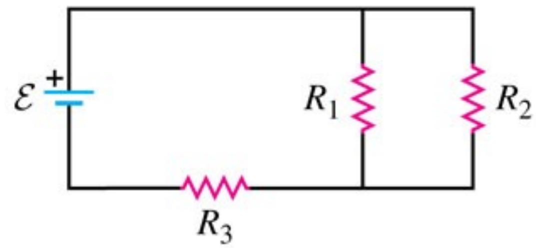
$6V = 3A \cdot r$

$\Rightarrow r = 2\Omega$

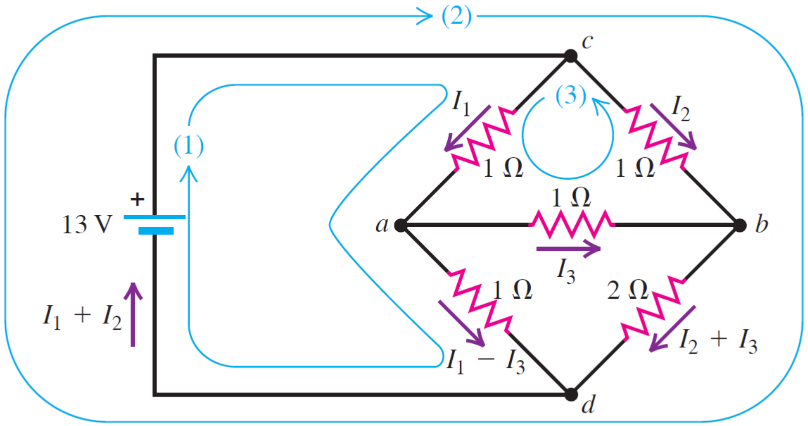
Example 3

$\varepsilon = 8.0\ V, R_1 = 3.0\Omega, R_3 = 2.0\Omega,$ & R_2 can vary between 3.0Ω and 6.0Ω.

1) For what value of R_2 is the power dissipated by R_1 the greatest?
2) What is that power?



Example 26.6 (If time permits)



Find the equivalent resistance for all five resistors.