ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 12: Current, Resistance, & Electromotive Force

Sep 25, 2024

Reminder: First in-class exam on Oct 2 8:00-8:50 am

Online review on Sep 30 (Next Monday office hour)

https://tennessee.zoom.us/j/85121460657

In Our Last Lecture

Electric Current: the NET amount of charge passing through a given area per unit time

$$I = \frac{dQ}{dt} = |q|nAv_d$$

Current Density ($J = I/A$) & **Vector Current Density** ($\vec{J} = qn\vec{v}_d$)

Resistivity ρ v.s. **Resistance R**: a measure of the movability of electrons in a given material

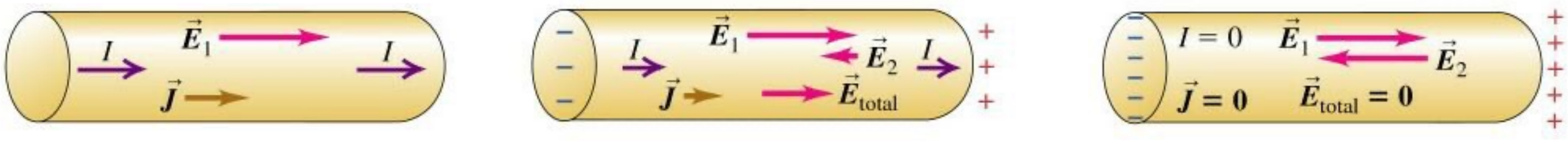
$$\rho = \frac{\vec{E}}{\vec{J}} = \frac{E}{J}, \qquad R = \frac{\rho L}{A}$$

Ohm's Law: the voltage-current relation for ohmic resistors

$$V = IR$$

Temperature dependence: $R(T) = R_0[1 + \alpha(T - T_0)]$

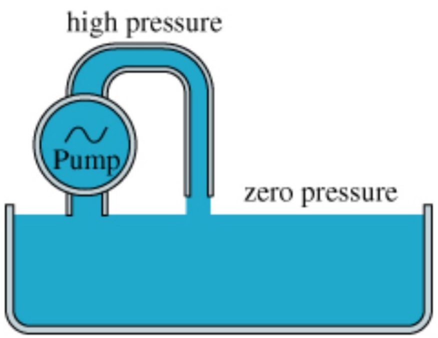
Steady Current



Electric field $\vec{E}_1$ produced inside conductor causes current

Current causes charge to build up at ends, producing opposing field $\vec{E}_2$ and reducing current

After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$: total field $\vec{E}_{total} = 0$ and current stops completely



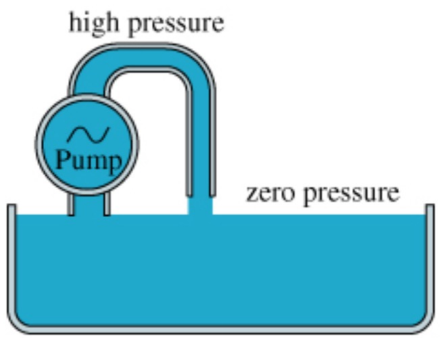
A steady current will exist in a conductor only if it is part of a complete circuit.

To maintain a steady current in an external circuit, we require the use of a source that supplies electrical energy.

The influence that makes current flow from lower to higher potential is called **electromotive force** (abbreviated emf and pronounced "ee-em-eff"), and a circuit device that provides emf is called a source of emf.

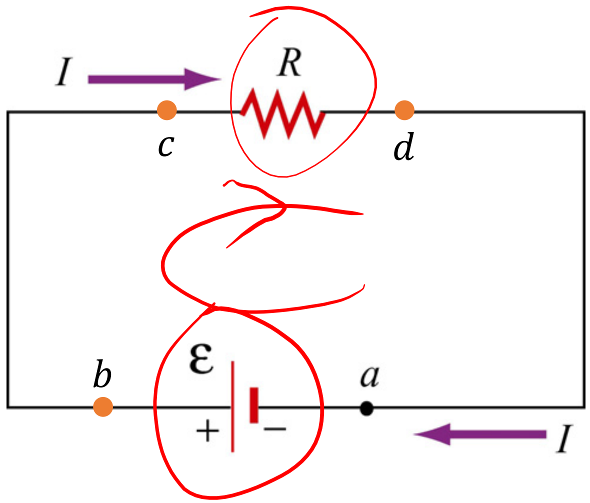
EMF

- "Electromotive force" is a poor term because emf is not a force but an energy-per-unit-charge quantity, like potential.
- The SI unit of emf is the same as that for potential, the volt 1V=1J/C
- A typical flashlight battery has an emf of 1.5 V; this means that the battery does 1.5 J of work on every coulomb of charge that passes through it.
- The symbol for emf is $\mathcal{E}$ (a script capital E).



Complete Circuit

A complete circuit is made by connecting the resistance R to the terminals of a source (emf) with a conducting wire.



$I = \frac{\varepsilon}{R}$

$$V_{ba} = V_b - V_a = \varepsilon$$

$$V_{cb} = 0$$

$$V_{dc} = V_d - V_c = -IR \quad \text{(Ohm's law)}$$

$$V_{da} = 0$$

If I go around the circuit and come back to the same point,
THE VOLTAGE MUST BE THE SAME!

$$0 = V_{aa} = (V_a - V_d) + (V_d - V_c) + (V_c - V_b) + (V_b - V_a) = \varepsilon - IR = 0 \quad \text{(Counterclockwise)}$$

If I count in a clockwise way,
$0 = V_{aa} = -\varepsilon + IR = 0.$

The direction of completing the voltage loop does not matter.

Internal Resistance of EMF

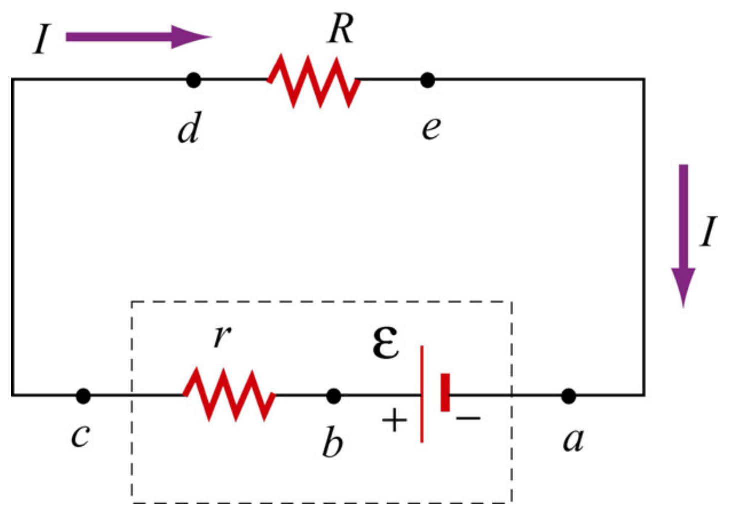
Internal resistance is an intrinsic part of a source of emf.

$$\varepsilon - Ir - IR = 0$$

Terminal voltage $V_{ca} = \varepsilon - Ir <$ emf

$$I = \frac{\varepsilon}{R + r}$$

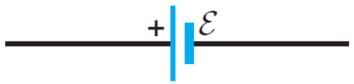
Current equals the source emf divided by the **total** circuit resistance $R + r$.



Symbols for Circuit Diagram

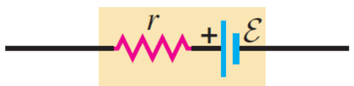
Conductor with negligible resistance

Resistor

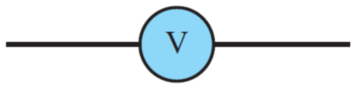


Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)

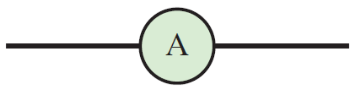
or



Source of emf with internal resistance r (r can be placed on either side)



Voltmeter (measures potential difference between its terminals) $R_V = \infty$



Ammeter (measures current through it) $R_A = 0$

Example 1

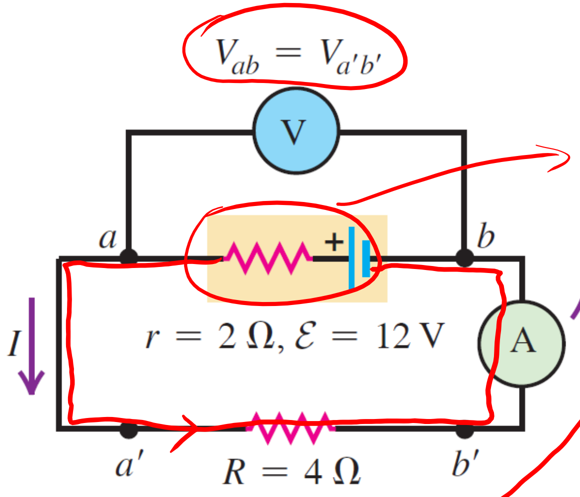
Find the voltmeter reading V_{ab} & ammeter reading I.

$$\mathcal{E} - Ir - IR = 0$$

$$\Rightarrow \quad I = \frac{\mathcal{E}}{R+r} = \frac{12\,V}{2\Omega + 4\Omega} = 2\ V/\Omega = 2\ A$$

$$V_{ab} = \mathcal{E} - Ir = 12V - 2A \times 2\Omega = 8\ V$$

$$V_{ab} = V_{a'b'} = IR = 2A \times 4\Omega = 8V$$



$V_{ab} = \mathcal{E} - Ir$

$\oplus = 0$

$\Downarrow$

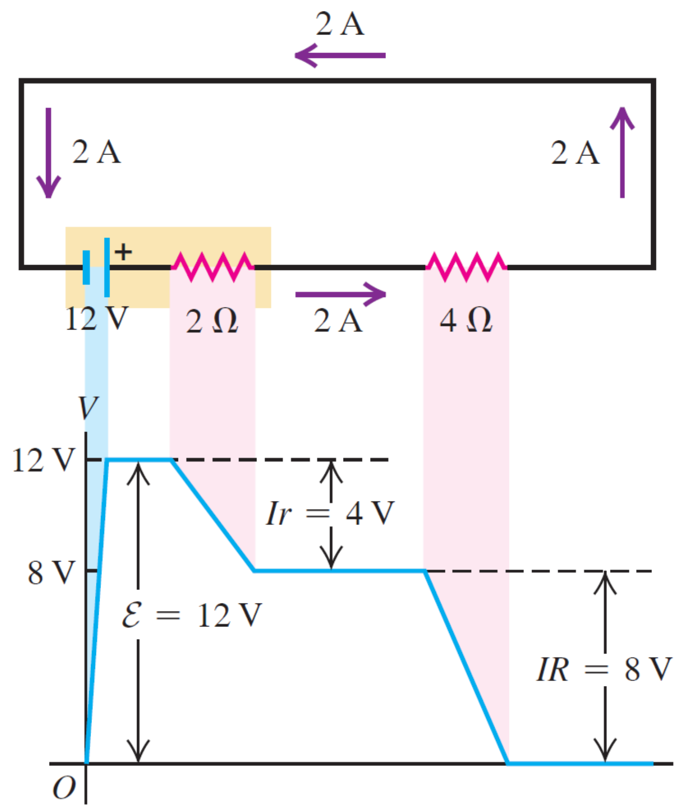
$-IR$

Potential Changes

The figure shows how the potential varies as we go around a complete circuit.

The potential rises when the current goes through a battery, and drops when it goes through a resistor.

Going all the way around the loop brings the potential back to where it started.



Power of Electric Circuit

Power delivered to or extracted from a circuit element

$$P = V_{ab} I$$

Voltage across circuit element

Current in circuit element

Power is the energy transfer (or work) per unit time: $P = \frac{d\Delta E}{dt}$

$$\frac{d\Delta E}{dt} = V_{ab} I = V_{ab} \frac{dq}{dt} = \frac{d(qV_{ab})}{dt} \Rightarrow \Delta E = qV_{ab}$$, the electric potential energy

Unit of Power is $\left(\frac{1J}{C}\right)\left(\frac{1C}{s}\right) = 1\frac{J}{s} = 1W$

Power delivered to a resistor

$$P = V_{ab} I = I^2 R = \frac{V_{ab}{}^2}{R}$$

Voltage across resistor

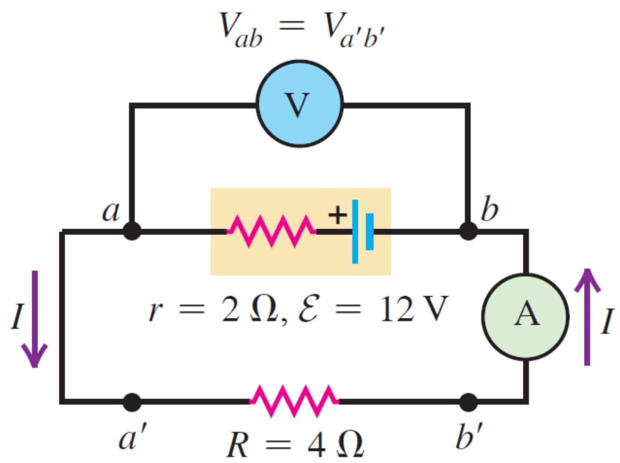
Current in resistor

Resistance of resistor

Example 2

Find the rates of energy conversion (chemical to electrical) and energy dissipation in the battery, the rate of energy dissipation in the 4 Ω resistor, and the battery's net power output.

Rates of energy conversion = Power



$$P_{battery} = I\varepsilon =$$

$$P_{internal} = I^2 r =$$

$$P_{resistor} = I^2 R =$$

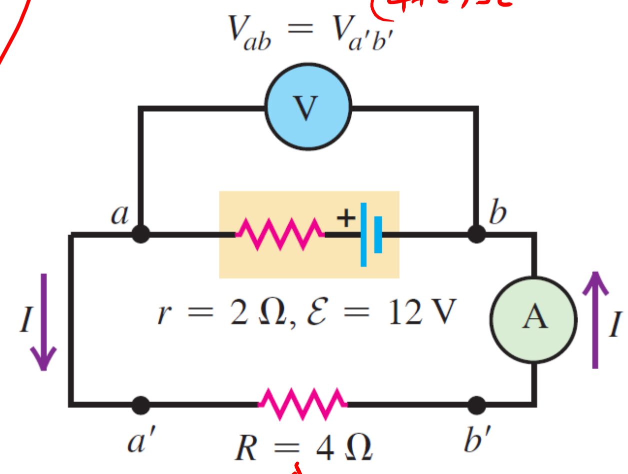
Example 2

Power of Internal Resist.

Find the rates of energy conversion (chemical to electrical) and energy dissipation in the battery, the rate of energy dissipation in the 4 Ω resistor, and the battery's net power output.

Rates of energy conversion = Power

$\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R+r} = \frac{12V}{(4+2)\Omega} = 2A$



$P_{battery} = I\varepsilon = 2A \times 12V = 24W$

$P_{internal} = I^2 r = (2A)^2 \times 2\Omega = 8W$

$P_{resistor} = I^2 R = (2A)^2 \times 4\Omega = 16W$

$= P_{bat.} - P_{int.} = 24W - 8W = 16W$

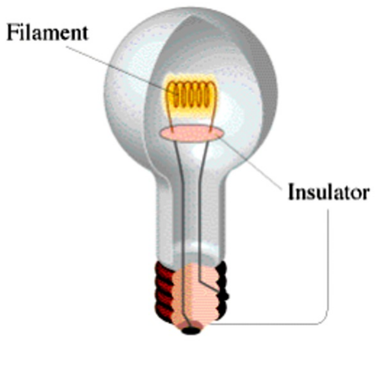
Power of Resistor

Clicker question

Two light bulbs operate at 120 V, but one has a power rating of 25 W while the other has a power rating of 100 W. Which one has the greater resistance?

A. The 25W bulb
B. The 100W bulb
C. The same
D. Not enough information

320950



$$P = VI = I^2R = \frac{V^2}{R}$$

$$R = \frac{V^2}{P}$$

Two space heaters in your living room are operated at 120 V. Heater 1 has twice the resistance of heater 2. Which one will give off more heat?

A. Heater 1
B. Heater 2
C. The same

$$P = VI = I^2R = \frac{V^2}{R}$$

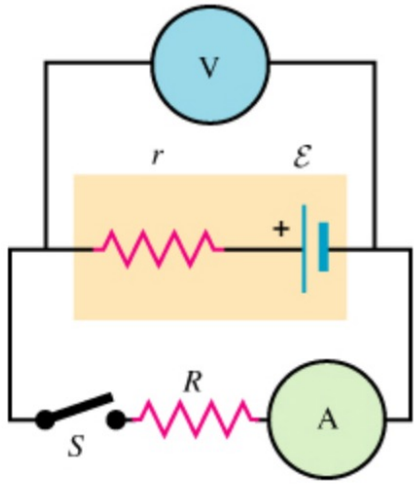
Example 3

When switch S in the figure below is open, the voltmeter V of the battery reads $3.05V$. When the switch is closed, the voltmeter reading drops to $2.95V$, and the ammeter A reads $1A$. Assume that the two meters are ideal, so they don't affect the circuit.

Find emf of the battery ε.

Find the internal resistance of the battery r.

Find the circuit resistance R



Example 3

Ⓥ is measuring emf because $I=0$

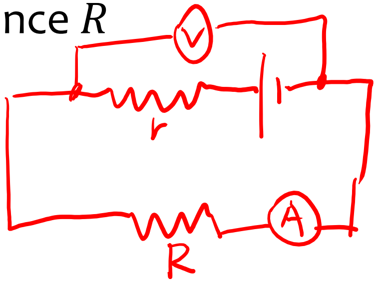
When switch S in the figure below is open, the voltmeter V of the battery reads $3.05V$. When the switch is closed, the voltmeter reading drops to $2.95V$, and the ammeter A reads $1A$. Assume that the two meters are ideal, so they don't affect the circuit.

Find emf of the battery ε. $= 3.05\,V$

Find the internal resistance of the battery r.

Find the circuit resistance R

When S closed



$$\overset{3.05V}{\mathcal{E}} \underbrace{- Ir}_{2.95V} - IR = 0$$

$$3.05V - \underset{1A}{I}\,r = 2.95\,V \Rightarrow r = \frac{(3.05-2.95)V}{1A} = 0.1\Omega$$

$$1A \cdot R = 2.95V$$

$$\Rightarrow R = 2.95\Omega$$