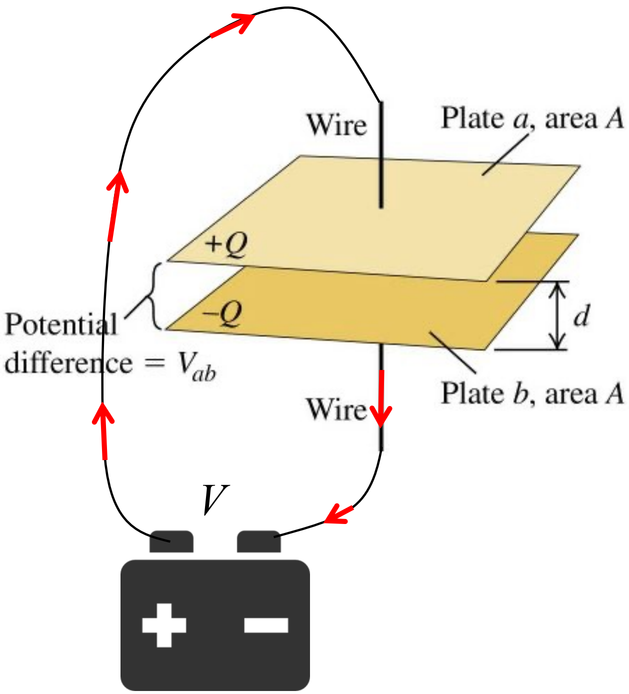
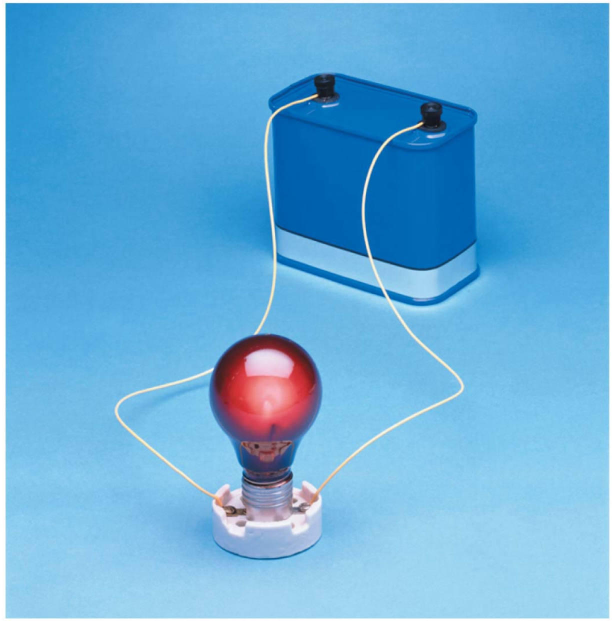
ELECTRICITY AND MAGNETISM (PHYS 231)

Lecture 11: Current, Resistance, & Electromotive Force

Sep 23, 2024

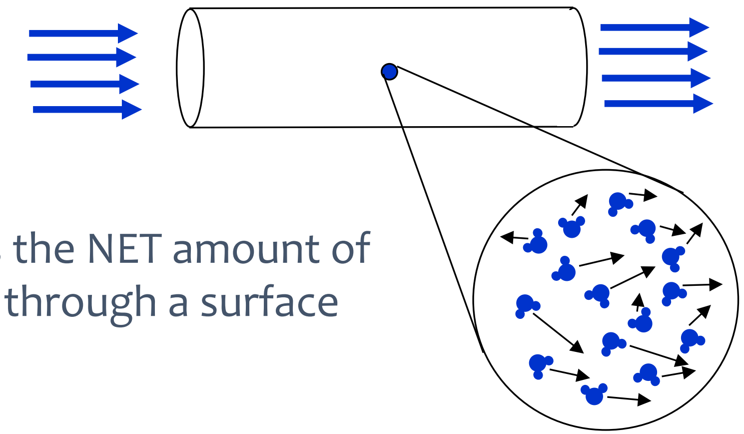
Dynamics of Charging





Water Flow v.s. Electric Current

H_20
gallons/minute

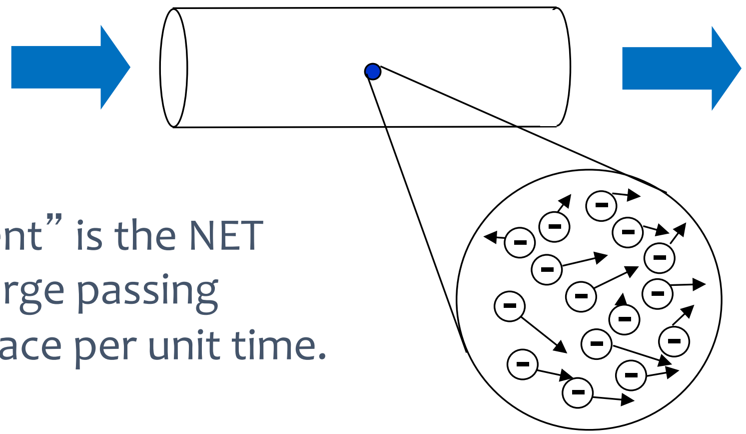


“Flow Rate” is the NET amount of water passing through a surface per unit time.

Individual molecules are bouncing around with speeds of km/s!

Net water velocity is m/s

I
Coulombs/s



“Electric Current” is the NET amount of charge passing through a surface per unit time.

Individual electrons are bouncing around with very high speed ($10^6 m/s$)

Electron “drift” velocity may be $10^{-4} m/s$

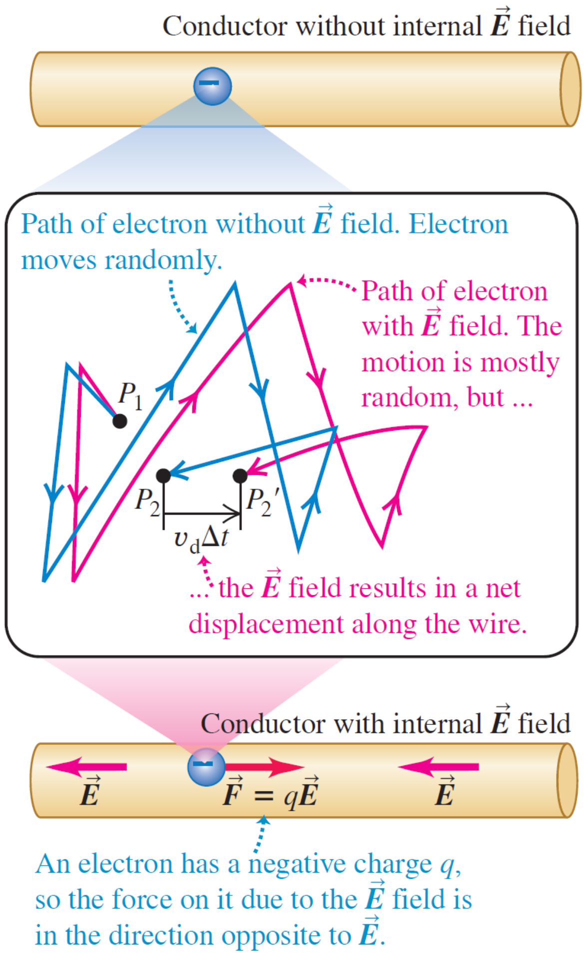
Beyond the Electrostatic Limit

Free electrons move randomly inside an electrostatic conductor (vanishing internal electric field).

Random motions of electrons do not generate any net flow of charge in any direction (no current).

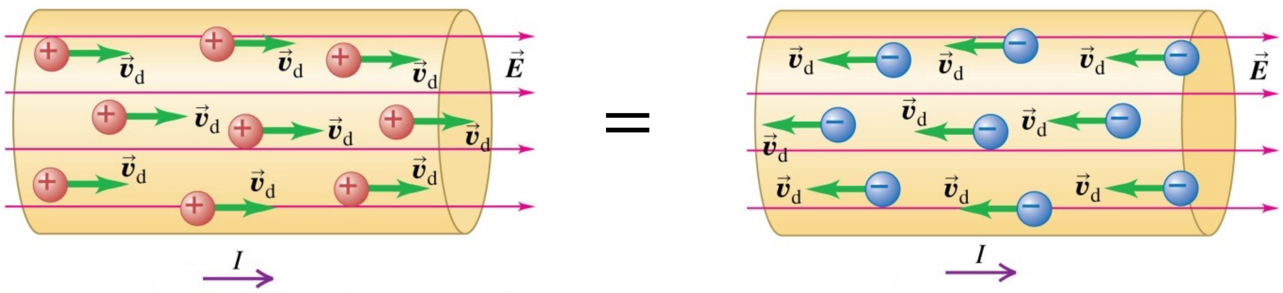
However, if a constant, steady electric field $\vec{E}$ is established inside a conductor…

Electrons start to “drift” as a whole with a drift velocity $\vec{v}_d$ (finite current).



Electric Current

- Electric current I has a direction.
- If the moving charges are **positive**, directions of current & charges are the **SAME.**
- If the moving charges are **negative**, direction of current & charges are the **OPPOSITE**.



The direction of I aligns with that of the internal electric field $\vec{E}$.

Electric Current is the NET amount of charge passing through a given area per unit time.

For example, if a net charge dQ flows through an area in a time dt, the current *I* through the area

$$I = \frac{dQ}{dt}$$

Electric Current

Electric Current is the NET amount of charge passing through a given area per unit time.

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$$I = \frac{dQ}{dt}$$

Unit of current is $1\ Ampere\ =\ 1\ Coulomb/1\ second$

$$A = \frac{C}{s}$$

Current I is **NOT** a vector!

André-Marie Ampère
(1775 - 1836)

Microscopic Expression of I

Suppose there are n moving charged particles per unit volume passing through a conductor with a cross section area A. The drift velocities of the charges are assumed to be the same with magnitude v_d. Calculate the current.

$$dQ = |q|(nAv_d dt)$$

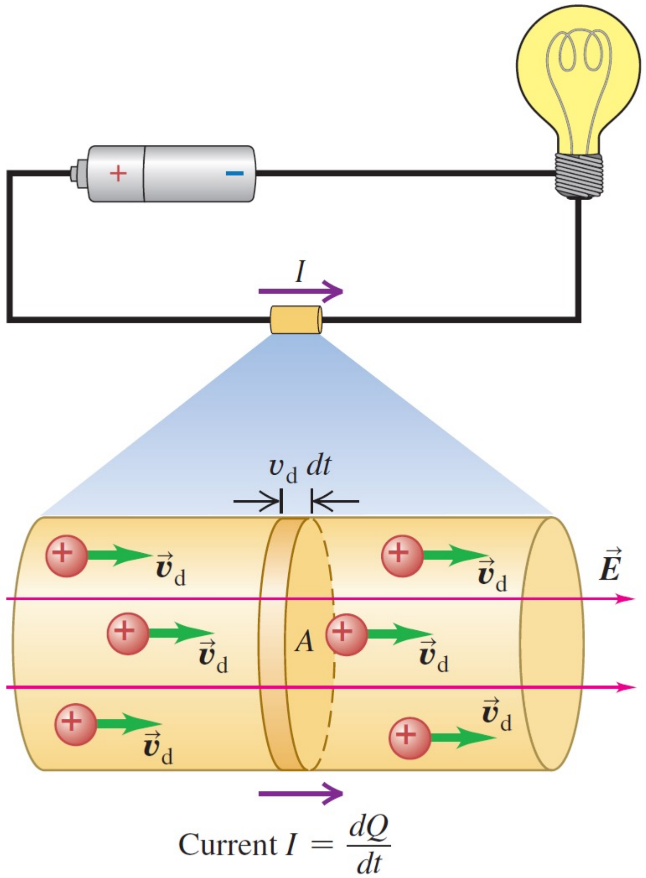
dQ is the total charge inside the volume $v_d dt$

$$I = \frac{dQ}{dt} = |q|nAv_d$$

Current Density (the current per cross-sectional area)

$$J = \frac{I}{A} = |q|nv_d$$

In unit of A/m^2



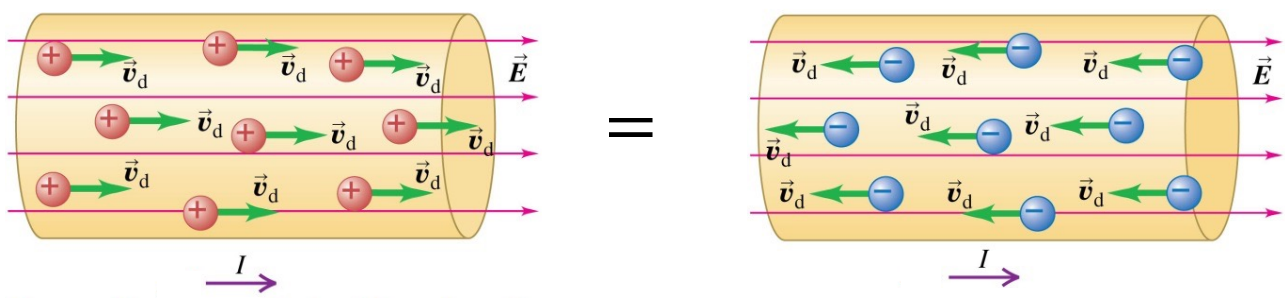
Promote J to a vector

Scalar Current Density

$$J = \frac{I}{A} = |q|nv_d$$

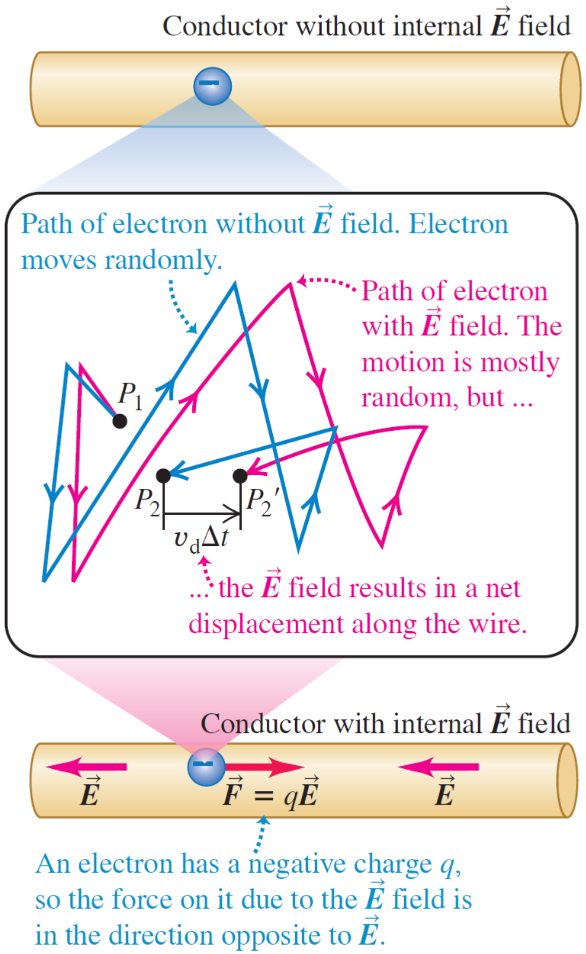
Vector Current Density

$$\vec{J} = qn\vec{v}_d$$



The direction of $\vec{J}$ is determined by the sign of the charges and the drift velocity, and is independent of whether we are using the positive or negative charges as the current carrier.

Drift Velocity Revisited



Why don't charges keep accelerating forever?

In real conductors, the charged particle will experience frequent collisions, into random directions, with the atoms compromising the material.

A balance is reached and the net overall motion becomes steady (i.e. all electrons reach the drift velocity v_d).

Just like running down a busy street

Resistivity

The overall collisions balance out the acceleration by the electric field. This effect is called the **resistivity** of the material.

In practice, the resistivity is quantified by

$$\rho = \frac{\vec{E}}{\vec{J}} = \frac{E}{J} \qquad \Omega \cdot m$$

The unit of ρ is $\frac{\left(\frac{V}{m}\right)}{\left(\frac{A}{m^2}\right)} = \left(\frac{V}{A}\right) \cdot m$

A new unit ohm (or Ω) is defined to be $1\Omega = V/A$.

The reciprocal of ρ is called "conductivity", usually denoted as σ.

For a given temperature, if ρ of a material does NOT depend on E, then this material is called "ohmic". Otherwise, it is nonohmic.

Conductors, Insulators, & Semiconductors

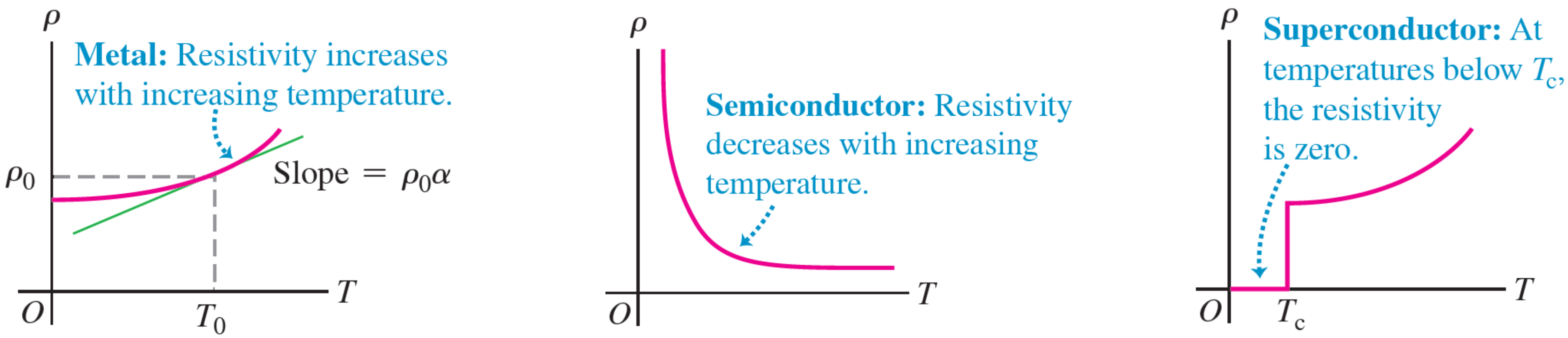
TABLE 25.1 Resistivities at Room Temperature (20°C)

	Substance	$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

- A perfect conductor has $\rho = 0$.
- A perfect insulator has $\rho = \infty$.

Resistivity is a measure of the capability for electrons to move within a certain material.

Temperature Dependence of ρ



Temperature dependence of resistivity:

Resistivity at temperature T

Temperature coefficient of resistivity

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

Resistivity at reference temperature T_0

Temperature dependence of ρ for metals

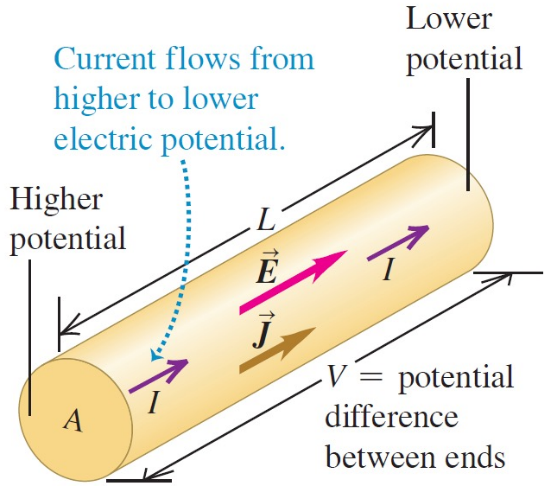
Resistance (*)

Assume the length of a conducting wire is L, then the potential difference across the wire is

$$V = EL$$

Since we also have

$$J = \frac{E}{\rho}, \qquad J = \frac{I}{A}$$



➡ $$V = EL = \rho JL = \frac{\rho L}{A} I$$

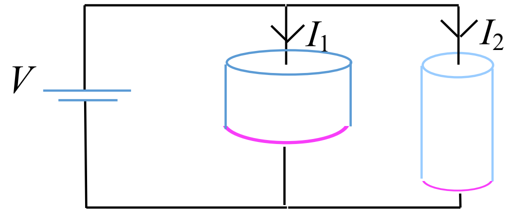
➡ Ohm's Law: $V = IR$ Resistance: $R = \frac{\rho L}{A}$

Resistance & Ohm's Law (*)

Ohm's Law: $V = IR$ Resistance: $R = \frac{\rho L}{A}$

- The resistance is proportional to the length L of the material and inversely proportional to the cross section area A.
- The unit for resistance is Ohm (Ω).
- Resistivity ρ is an intrinsic property of a certain type of material (copper, steel, water,…). It does NOT depend on the geometry of the object.
- Resistance R is a property of a specific object (e.g. a given piece of wire with certain geometry dimensions). It does depend on the geometry.
- Apparently, $R(T) = R_0[1 + \alpha(T - T_0)]$

Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has *twice the length* of R_1 but *half the radius* of R_1. These resistors are then connected to a battery V as shown:



What is the relation between I_1, the current flowing in R_1, and I_2, the current flowing in R_2?

The resistivity of both resistors is the same (*r*). Therefore the resistances are related as:

$$R_2 = \rho \frac{L_2}{A_2} = \rho \frac{2L_1}{(A_1/4)} = 8\rho \frac{L_1}{A_1} = 8R_1$$

The resistors have the same voltage across them; therefore

$$I_2 = \frac{V}{R_2} = \frac{V}{8R_1} = \frac{1}{8} I_1$$