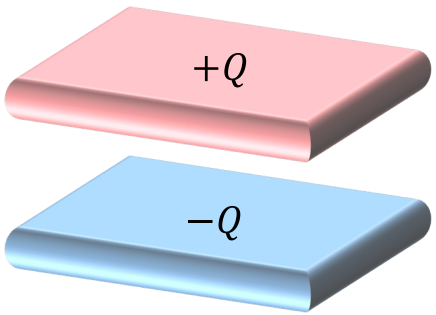
ELECTRICITY AND MAGNETISM (PHYS 231)

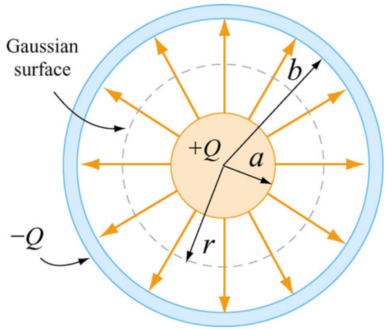
Lecture 10: Capacitance & Dielectrics

Sep 18, 2024

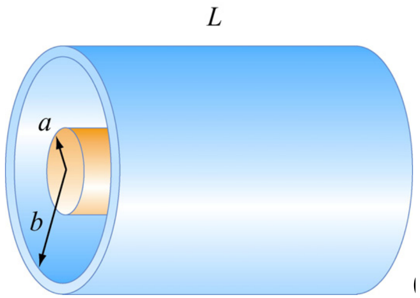
Capacitors



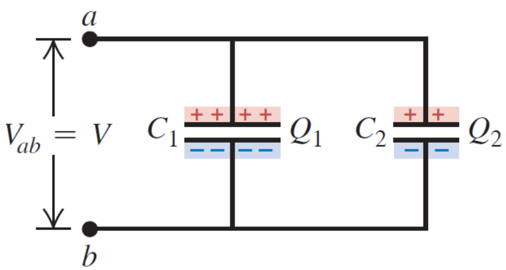
Parallel-plate Capacitor

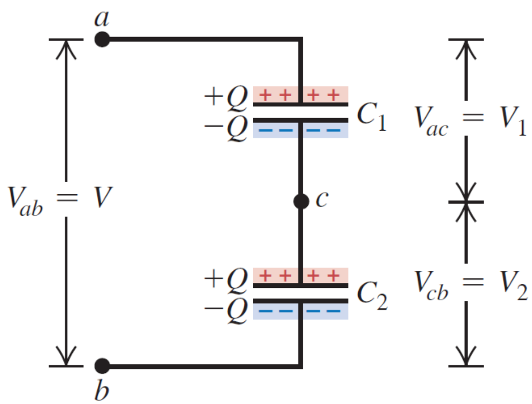


Spherical Capacitor



Cylindrical Capacitor





Energy Storage in Capacitors

Potential energy U of a charged capacitor = the work W required to charge it

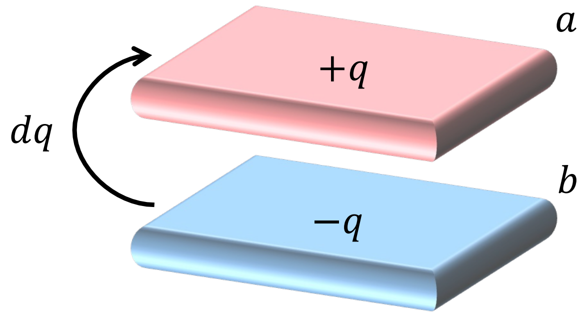
Starting from a capacitor C with charge q and voltage v, we have $C = q/v$.

Consider adding dq charge to the capacitor (i.e. bring dq from b to a), then the work done during this process is

$$dW = vdq = \frac{qdq}{C}$$

If we charge the capacitor from zero charge to Q, the total work is

$$W = \int_0^W dW = \int_0^Q \frac{qdq}{C} = \frac{Q^2}{2C}$$



Potential energy stored in a capacitor $\rightarrow$ $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

Magnitude of charge on each plate (Q)

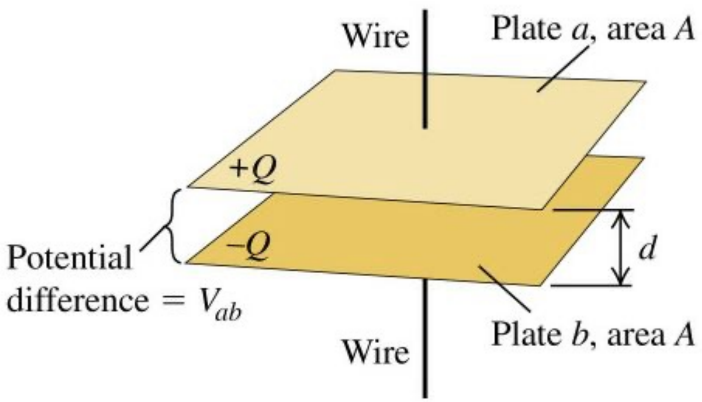
Capacitance (C)

Potential difference between plates (V)

Three equivalent expressions for U.
Pick one that fits the problem better.

Parallel Plate Capacitors

$$U = \frac{1}{2} C V^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{Ad\varepsilon_0 E^2}{2}$$



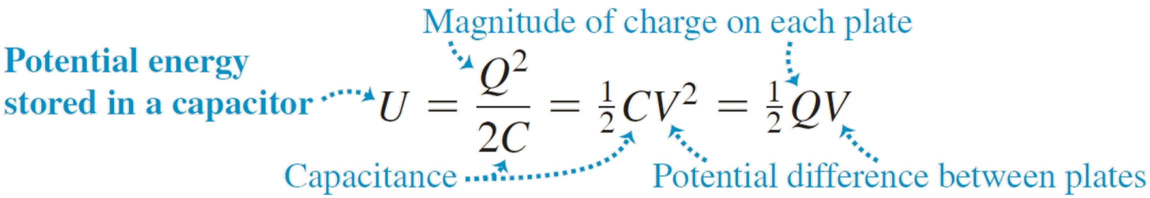
Ad is the volume of space sandwiched between the plates.

Electric energy density in a vacuum

$$u = \frac{U}{volume} = \frac{1}{2}\varepsilon_0 E^2$$

- The above expression holds for any electric field configuration in vacuum.
- Electric field by itself has energy.
- This is the same electric potential energy we have met before.

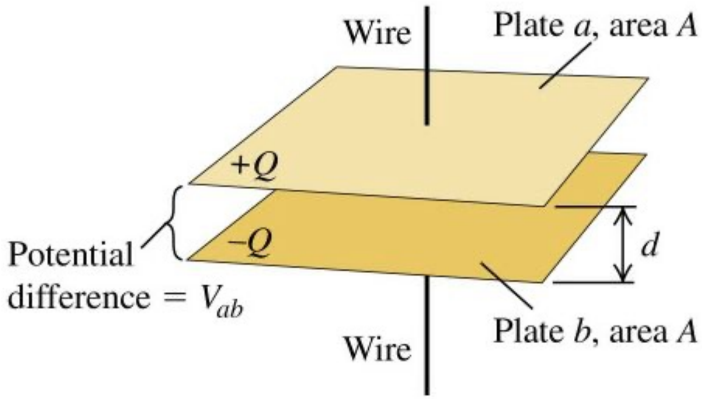
How to Enhance Capacitance?



For a fixed voltage V, a larger C means a greater energy storage.

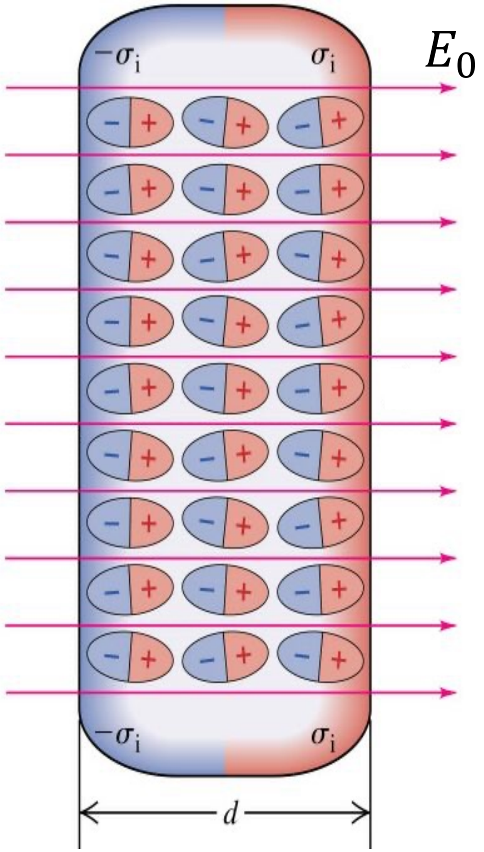
For parallel plate capacitors, how to enhance $C = \epsilon_0 A/d$?

1. Increase area A;
2. Reduce distance d;
3. Insert dielectric materials.



In principle, all insulators are dielectric materials…
But we want the good ones!

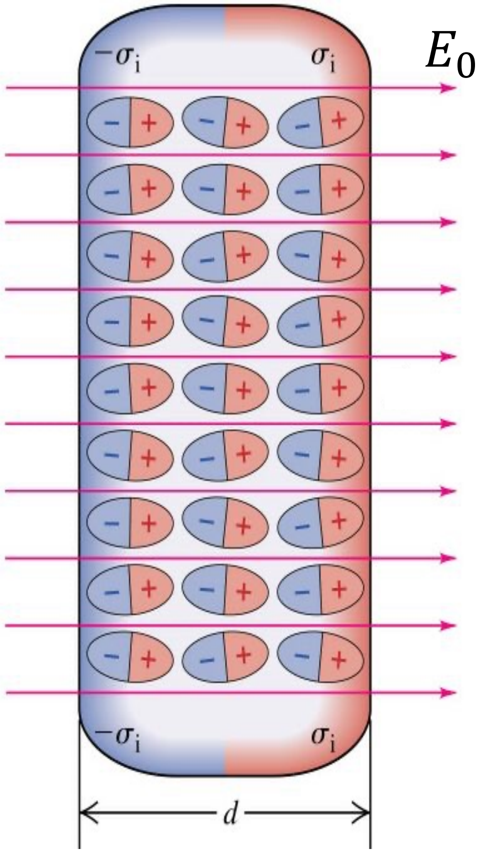
Insulators Revisited



1. Electrons inside an insulator cannot move freely.
2. When applying an electric field E_0, atoms in insulators will be polarized. Namely, a local separation between the electron & the ion will be created.
3. Polarized atoms act as small dipoles, creating local electric field opposite to the external electric field.
4. Electric field E inside an insulator is always smaller than E_0.
5. The ratio K between E_0 and E is the **dielectric constant** of the insulator.

$$K = \frac{E_0}{E}$$

Insulators Revisited



$$K = \frac{E_0}{E} \geq 1$$

Dielectric Constant

Remarks

1. Dielectric constant is an intrinsic property of material. It tells us how much an external electric field is reduced inside the material compared with vacuum.
2. Vacuum has $K = 1$.
3. General insulators has $K > 1$.
4. Conductors can be viewed as a very special "insulator" with $K = \infty$. Why?

Electric field inside an electrostatic conductor is zero everywhere.

How does Dielectric Material Help Us?

Vacuum:

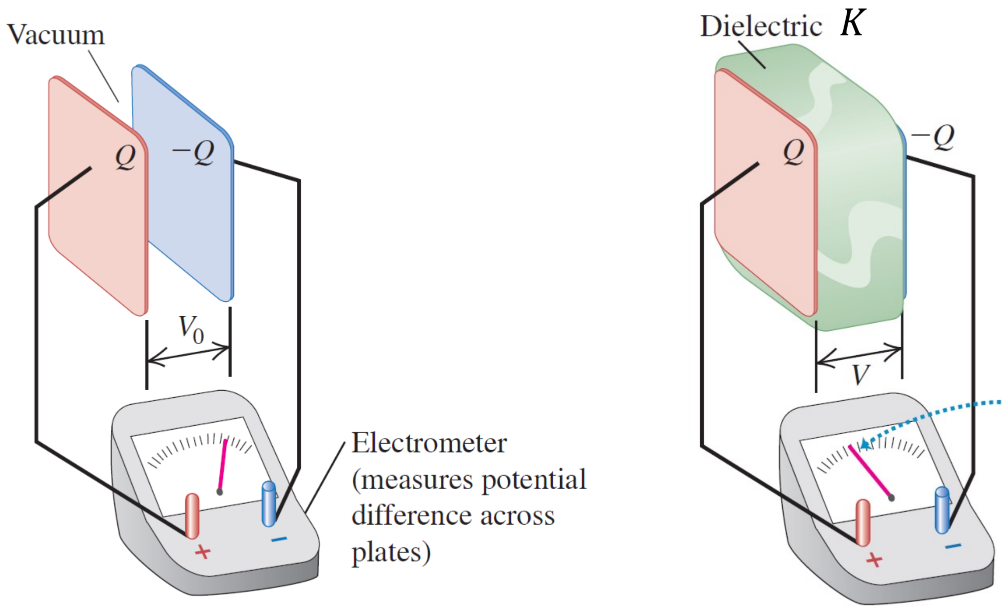
$$C_0 = \frac{Q}{V_0}$$

Dielectric material:

$$E = E_0/K \quad \rightarrow \quad V = V_0/K$$

$$C = \frac{Q}{V_0} = K\frac{Q}{V_0} = KC_0$$

$$C = KC_0$$



Inserting dielectric material with a dielectric constant of K will increase the capacitance by a factor of K!

We need insulators with a high dielectric constant!

How does Dielectric Material Help Us?

Material	ε_r
Vacuum	1 (by definition)
Air	1.000 589 86 ± 0.000 000 50 (at STP, for 0.9 MHz),[1]
PTFE/Teflon	2.1
Polyethylene/XLPE	2.25
Polyimide	3.4
Polypropylene	2.2–2.36
Polystyrene	2.4–2.7
Carbon disulfide	2.6
Mylar	3.1[2]
Paper	3.85
Electroactive polymers	2–12
Mica	3-6[2]
Silicon dioxide	3.9 [3]
Sapphire	8.9–11.1 (anisotropic) [4]
Concrete	4.5
Pyrex (Glass)	4.7 (3.7–10)
Neoprene	6.7[2]
Rubber	7
Diamond	5.5–10
Salt	3–15
Graphite	10–15
Silicon	11.68
Silicon nitride	7-8 (polycrystalline, 1MHz)[5][6]
Ammonia	26, 22, 20, 17 (−80, −40, 0, 20 °C)
Methanol	30
Ethylene glycol	37
Furfural	42.0
Glycerol	41.2, 47, 42.5 (0, 20, 25 °C)
Water	88, 80.1, 55.3, 34.5 (0, 20, 100, 200 °C) for visible light: 1.77
Hydrofluoric acid	83.6 (0 °C)
Formamide	84.0 (20 °C)
Sulfuric acid	84–100 (20–25 °C)
Hydrogen peroxide	128 aq–60 (−30–25 °C)
Hydrocyanic acid	158.0–2.3 (0–21 °C)
Titanium dioxide	86–173
Strontium titanate	310

High dielectric constant:

$$K > 7$$

Low dielectric constant:

$$K < 3.9$$

Calcium copper titanate ($CaCu_3Ti_4O_{12}$):

$$K \sim 10{,}000$$

Permittivity

Permittivity: $\epsilon = K\epsilon_0$

Parallel Plate Capacitor	Vacuum	Dielectric with $K > 1$
Electric Field	$E_0 = \dfrac{\sigma}{\varepsilon_0}$	$E = \dfrac{\sigma}{\varepsilon}$
Capacitance	$C_0 = \dfrac{\varepsilon_0 A}{d}$	$C = \dfrac{\varepsilon A}{d}$
Electric Energy Density	$u_0 = \dfrac{1}{2}\varepsilon_0 E^2$	$u = \dfrac{1}{2}\varepsilon E^2$

When inside a material, we simply **replace ε_0 with ε** for all the quantities and laws/principles that originally involve ε_0.

Coulomb's & Gauss's Laws inside a Dielectric Media

	Vacuum	Dielectric Media with ε
Coulomb's Law	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$	$\vec{E} = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\hat{r}$
Gauss's Law	$\oiint_A \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$	$\oiint_A \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon}$

When inside a material, we simply **replace ε_0 with ε** for all the quantities and laws/principles that originally involve ε_0.

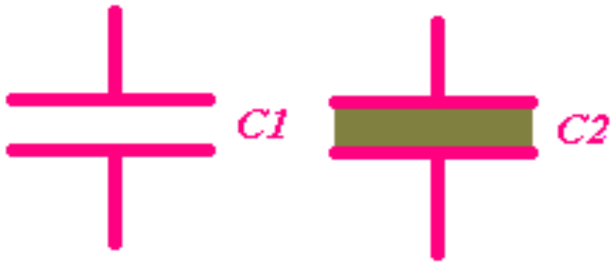
Two identical parallel plate capacitors are carrying the same amount of charge Q and remain disconnected from any battery. Then C_2 is filled with an insulator with $K > 1$. Compare the voltages of the two capacitors. Which one is correct?

✓ A. V1 > V2

B. V1 = V2

C. V1 < V2

D. Not enough information

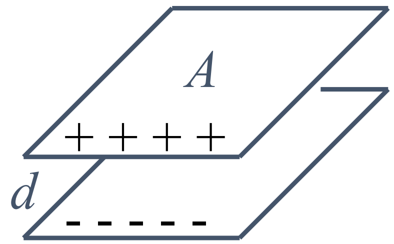


$$Q = CV$$

Example #1

A capacitor is charged to Q and then the battery is *disconnected*. Now we *pull the plates further apart* so that the final separation is $d_1 > d$

Which of the quantities *Q*, *C*, *V*, *U*, *E* change?



Q: Charge on the capacitor does not change.

C: Capacitance decreases.

V: Voltage increases.

U: Potential energy increases.

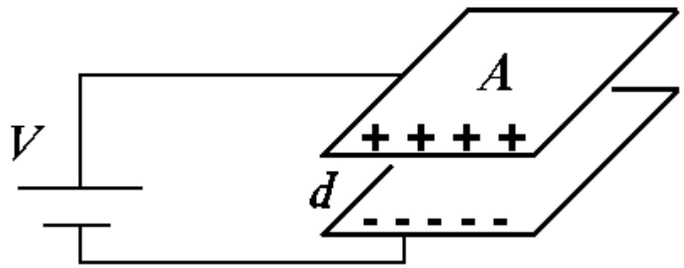
E: Electric field does not change.

$$C = \epsilon \frac{A}{d} \qquad V = \frac{Q}{C}$$

$$U = \frac{1}{2} QV \qquad E = \frac{\sigma}{\varepsilon}$$

Example #2

A capacitor is kept connected to a battery V. Now we *pull the plates further apart* so that the final separation is $d_1 > d$



Which of the quantities *Q*, *C*, *V*, *U*, *E* change?

Q: Charge decreases.

C: Capacitance decreases.

V: Voltage does not change.

U: Potential energy decreases.

E: Electric field decreases.

$$C = \epsilon \frac{A}{d} \qquad V = \frac{Q}{C}$$

$$U = \frac{1}{2} QV \qquad E = \frac{\sigma}{\varepsilon}$$