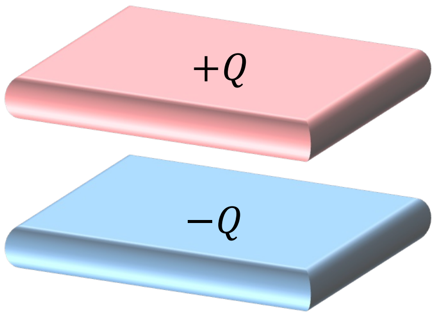
ELECTRICITY AND MAGNETISM (PHYS 231)

Lecture 8: Capacitance & Dielectrics

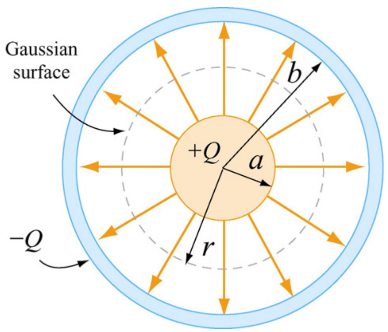
Sep 16, 2024

What is a Capacitor

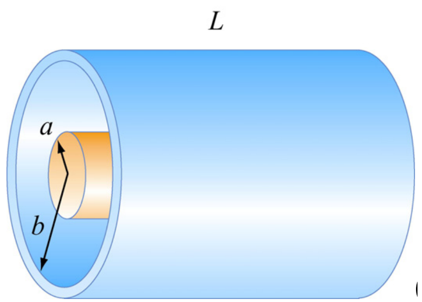
- A capacitor is a device that stores electric potential energy and electric charge.
- Any two conductors separated by an insulator (or vacuum) form a capacitor.
- Capacitors are charge neutral as a whole. When charges get transferred from one conductor to another, we are charging the capacitor.
- When we say that a capacitor has charge Q, or that a charge Q is stored on the capacitor, we mean that one conductor has charge +Q and the other has charge –Q.



Parallel-plate Capacitor



Spherical Capacitor



Cylindrical Capacitor

Application of Capacitors

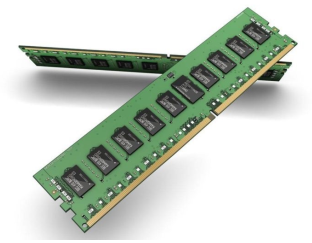
Energy Storage

Flash Photography

Sensing

Condenser Microphone

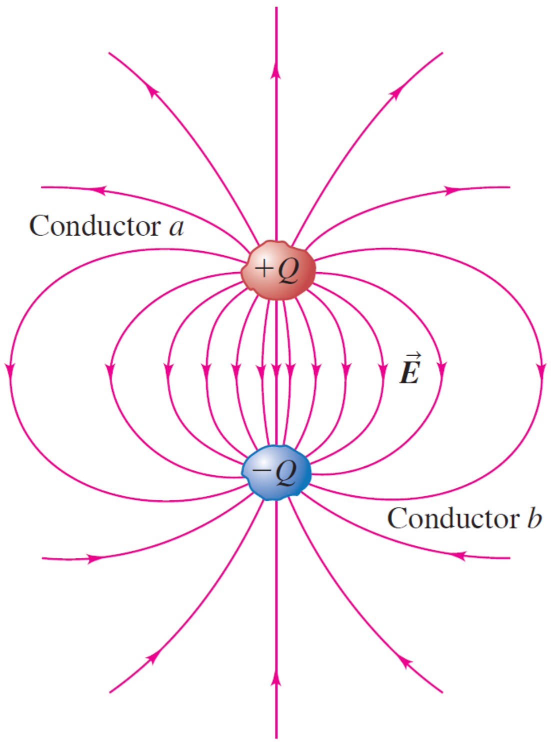
Signal Processing



DRAM

……

Capacitance



When charging a capacitor

1) Conductors get oppositely charged.

2) Conductors now have a potential difference

$$V_{ab} = V_a - V_b$$

3) As Q increases, V_{ab} increases proportionally and vice versa.

(e.g. if V_{ab} doubles, Q doubles as well.)

Capacitance of a capacitor

$$C = \frac{Q}{V_{ab}}$$

Q: Magnitude of charge on each conductor

V_{ab}: Potential difference between conductors (a has charge $+Q$, b has charge $-Q$)

Capacitance is a measure of the ability of a capacitor to store charge/energy.

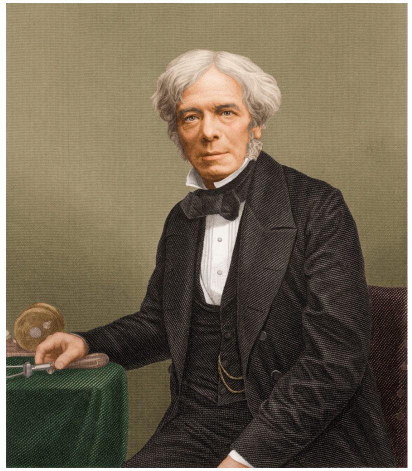
Capacitance

Capacitance of a capacitor $C = \dfrac{Q}{V_{ab}}$

- Q: Magnitude of charge on each conductor
- V_{ab}: Potential difference between conductors (a has charge $+Q$, b has charge $-Q$)

SI Unit of Capacitance is one Farad (1 F)

$1\text{F} = 1 \text{ farad} = 1\ C/V$ = 1 coulomb/volt



Michael Faraday
(1791 - 1867)

[We will talk about his law in a month…]

Example

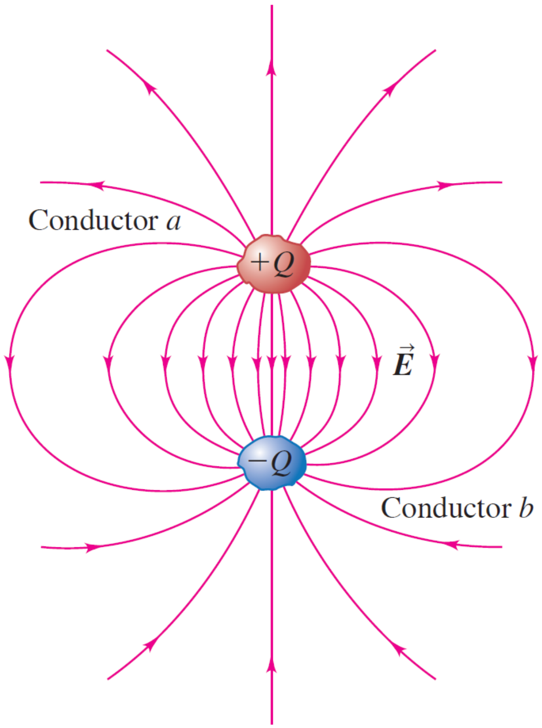
1) If we charge a 1F capacitor with a battery of 1V, the charge stored in the capacitor will be 1C.
2) If we know the charge stored within a 1F capacitor is 1C, then the potential difference between the two charged conductors will be 1V.

$$\textit{Capacitance is always positive}\quad C = \left|\frac{Q}{V_a - V_b}\right| = \frac{Q}{\Delta V}$$

(Q: Magnitude; ΔV: Magnitude)

The two conductors **a** and **b** are insulated from each other, forming a capacitor with a capacitance of C. We double the charge on **a** to $+2Q$ and the charge on **b** to $-2Q$. As a result of this change, the capacitance of the two conductors will become

A. 2C
B. 4C
C. C/2
D. C/4
✔ E. C



Recipe for Calculating Capacitance

Capacitance of a capacitor $\rightarrow$ $C = \dfrac{Q}{V_{ab}}$

- Q: Magnitude of charge on each conductor
- V_{ab}: Potential difference between conductors (a has charge $+Q$, b has charge $-Q$)

Step 1 Find the charge Q stored in the capacitor

Step 2 Find the potential difference ΔV between two conductors

Step 3 Calculate $Q/\Delta V$

Parallel Plate Capacitor

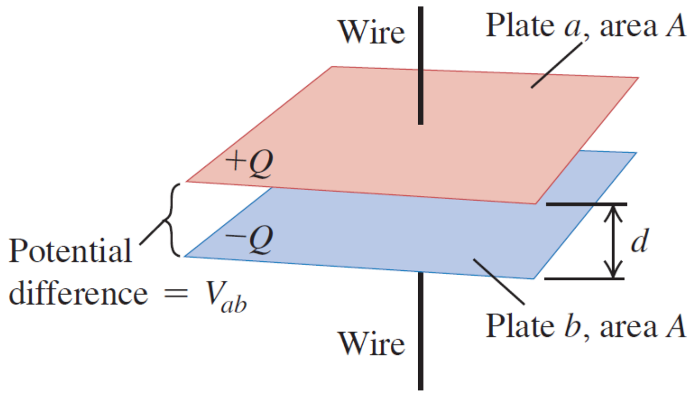
Step 2 Find ΔV: [i] Find electric field $\vec{E}$;
[ii] Integration of $\vec{E}$ to get ΔV

Using Gauss's law, the electric field is found to be uniform

$$\vec{E} = \frac{\sigma}{\varepsilon_0}(-\hat{y}) = \frac{Q}{A\varepsilon_0}(-\hat{y})$$

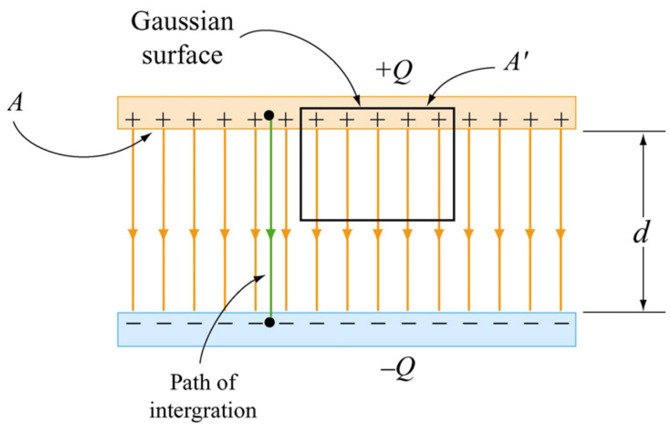
where the surface charge density $\sigma = Q/A$.

See Example 22.8



$$\Delta V = \int_a^b \vec{E} \cdot d\vec{l} = Ed = \frac{1}{\varepsilon_0}\frac{Qd}{A}$$

(Integral gets simplified due to the uniform E field)



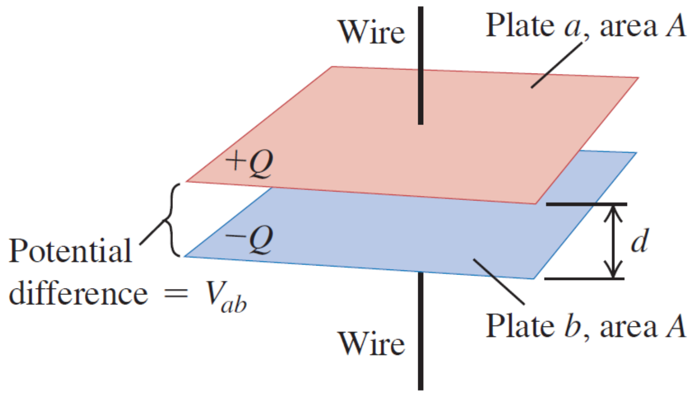
$$C = \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{d}$$

Capacitance of a parallel plate capacitor in vacuum

Parallel Plate Capacitor

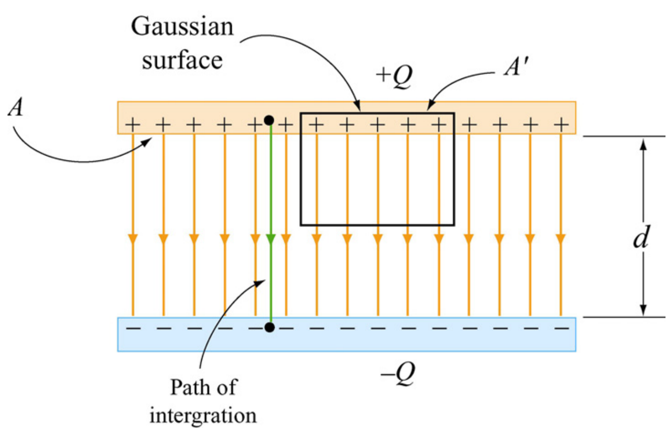
$$C = \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{d}$$

Capacitance of a parallel plate capacitor in vacuum



Some Remarks

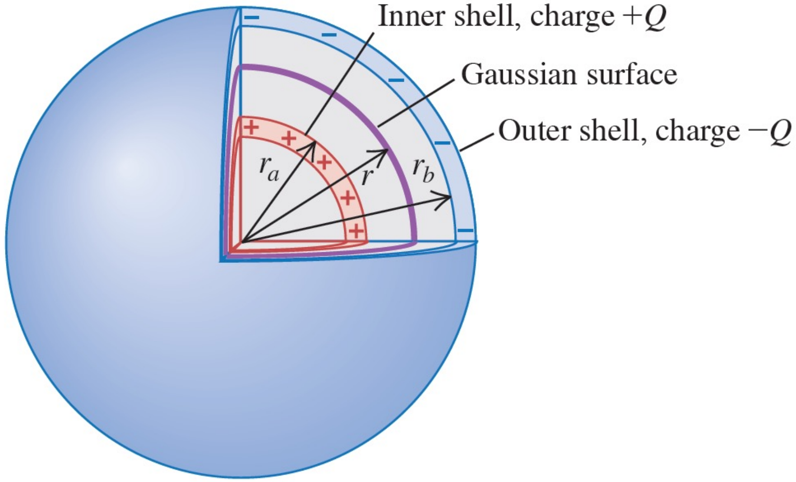
1) **ΔV is, by definition, ALWAYS positive.** If your calculation shows a negative ΔV, you probably encounter a sign error somewhere and will need to fix the sign of ΔV by taking its absolute value.
2) **Capacitance is, by definition, ALWAYS positive.**
3) **The capacitance only depends on the geometry of the capacitor**, i.e., C increases with area A and decreases with the distance d.
4) The above formula is valid only if the two conductors are separated by vacuum.



Spherical Capacitor

1) Using Gauss's law & a spherical Gaussian surface, the electric field is found to be

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



2) Calculate the voltage from r_a (positively charged sphere) to r_b (negatively charged sphere)

$$\Delta V = \int_{r_a}^{r_b} \vec{E}\cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

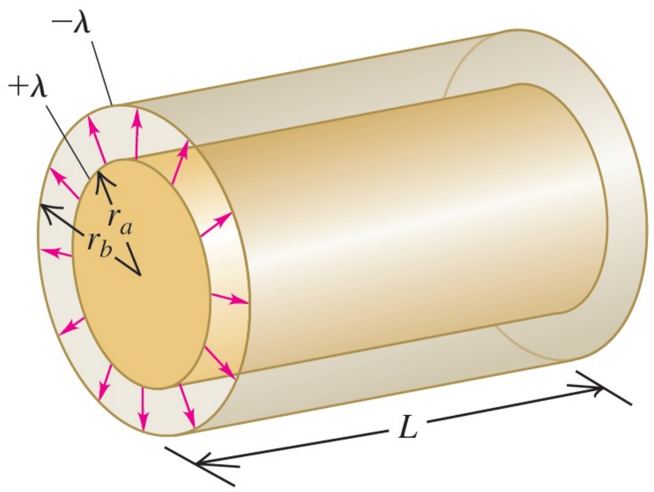
Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge $+Q$ and outer radius r_a, and the outer shell has charge $-Q$ and inner radius r_b.

3) Calculate $C = Q/\Delta V$

$$C = 4\pi\varepsilon_0 \frac{r_a r_b}{r_a - r_b}$$

The capacitance only depends on the geometry of the capacitor.

Cylindrical Capacitor



See Example 24.4 in the textbook for details.

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_b}{r_a}\right)}$$

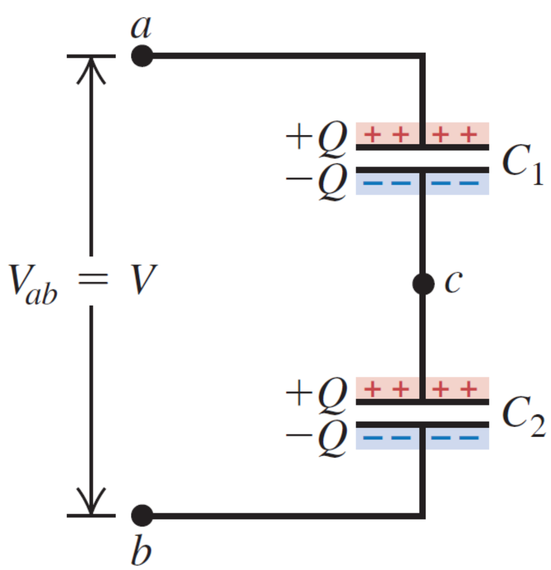
The capacitance only depends on the geometry of the capacitor.

Capacitors in the Real World

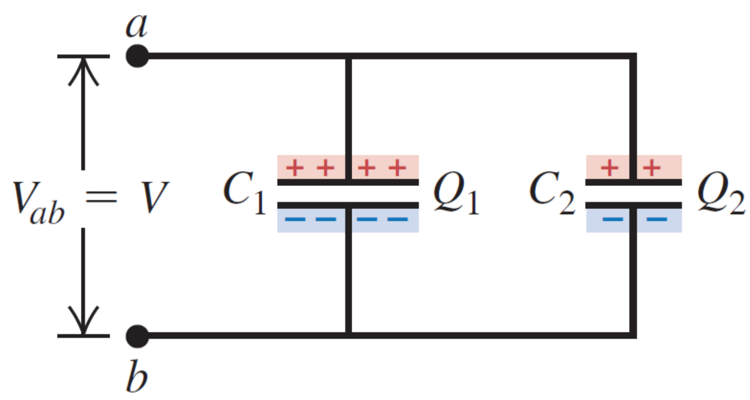
Symbol of capacitors in a circuit diagram is

Combining Capacitors

- Capacitors are manufactured with certain standard capacitances and working voltages. However, these standard values may not be the ones we actually need in a particular application.
- Combining standard capacitors to get an equivalent capacitor with our desired value.



Capacitors in Series



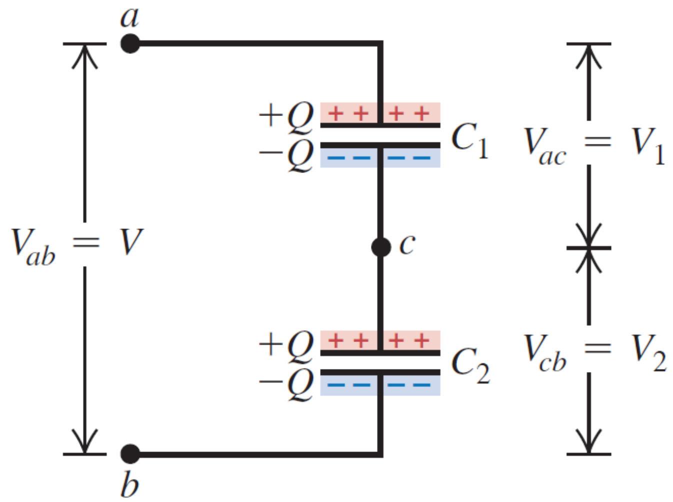
Capacitors in Parallel

Capacitors in Series

Fact #1: The electric potential along an ideal conducting wire is the same everywhere unless it hits a capacitor/resistor/…

Fact #2: **Two capacitors in series will store the same amount of charge Q when a voltage V_{ab} is applied.**

Fact #3: $V_{ab} = V_{ac} + V_{cb}$ or $V = V_1 + V_2$



$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C_{eq}}$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \quad \text{or} \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

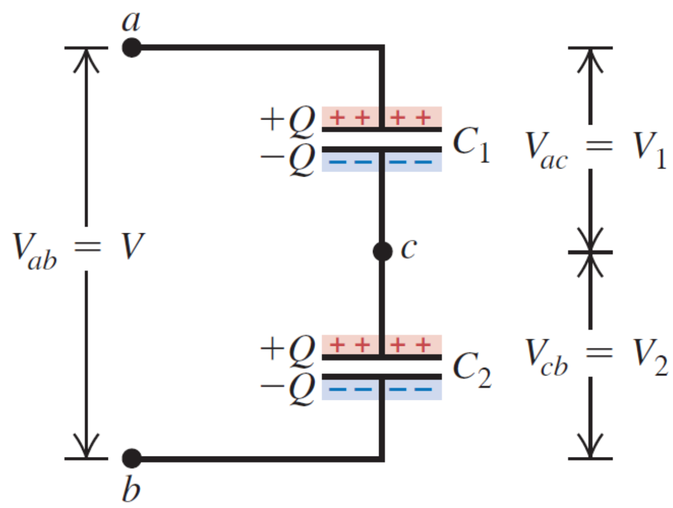
Equivalent capacitance for two capacitors in series

Capacitors in Series

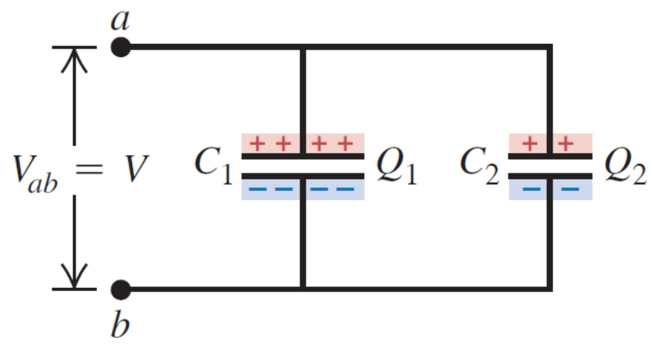
Equivalent capacitance for multiple capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum_i \frac{1}{C_i}$$

C_{eq} for capacitors in series is always **smaller** than each individual capacitor.



Capacitors in Parallel



Fact #1: The electric potential along an ideal conducting wire is the same everywhere unless it hits a capacitor/resistor/…

Fact #2: **Two capacitors in parallel share the same voltage V_{ab}.**

Fact #3: $V_{ab} = V = V_1 = V_2$

Fact #4: The charges stored in two parallel capacitors do NOT need to be the same.

The total charge stored in both capacitors is

$$Q = Q_1 + Q_2 = C_1V_1 + C_2V_2 = (C_1 + C_2)V = C_{eq}V$$

$$C_{eq} = C_1 + C_2$$

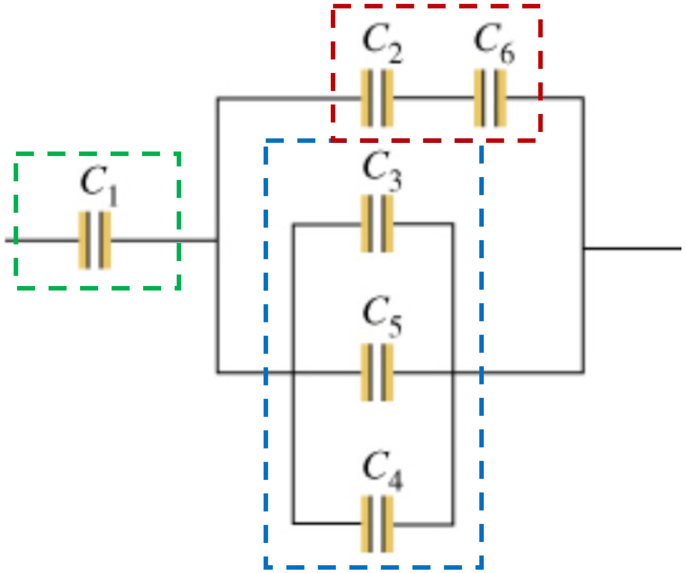
Equivalent capacitance for two capacitors in parallel

$$C_{eq} = \sum_i C_i$$

Equivalent capacitance for multiple capacitors in parallel

C_{eq} for capacitors in parallel is always **larger** than each individual capacitor.

Example: Find C_{eq} for the Capacitance Network



Step 1 Decompose the network into "blocks" of capacitors

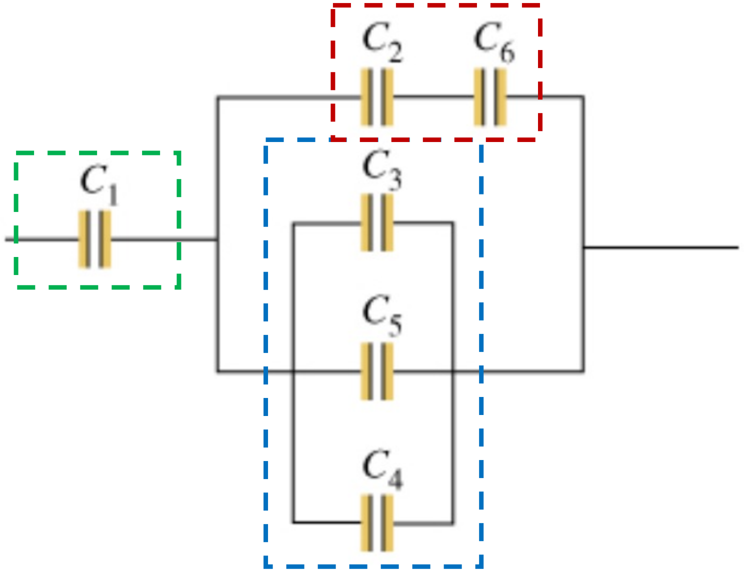
1) Blue block contains three capacitors in parallel
2) Red block contains two capacitors in series
3) Green block contains one capacitor

$$C_{green} = C_1, \qquad C_{blue} = C_3 + C_4 + C_5, \qquad C_{red} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_6}}$$

Example: Find C_{eq} for the Capacitance Network

Step 2 Identify relations among blocks

1) Blue & Red blocks are in parallel
2) C_{green} is in series with the combination of C_{blue} & C_{red}



$$C' = C_{blue} + C_{red} = C_3 + C_4 + C_5 + \frac{1}{\frac{1}{C_2} + \frac{1}{C_6}}$$

$$C_{eq} = \frac{1}{\frac{1}{C_{green}} + \frac{1}{C'}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_3 + C_4 + C_5 + \frac{1}{\frac{1}{C_2} + \frac{1}{C_6}}}}$$