ELECTRICITY AND MAGNETISM (PHYS 231)

Lecture 7: Electric Potential

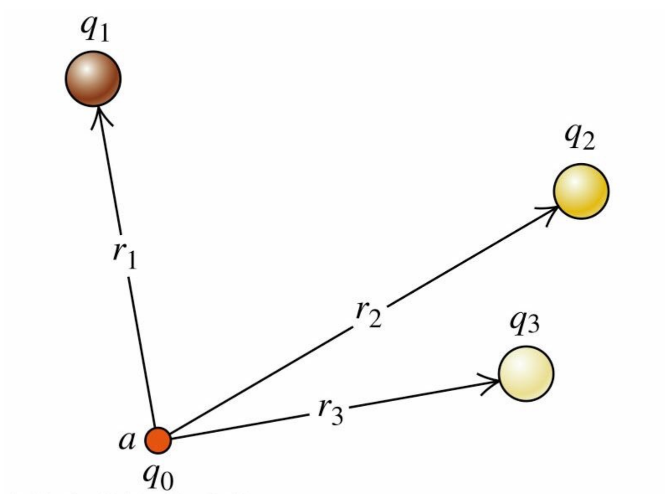
Sep 11, 2024

Carrying a Stone to the Top of a Pyramid

Given several charges $q_1, q_2, \ldots$ in place. Now a test charge q_0 is brought into position a.

We calculate the potential energy of q_0 with respect to each of the other charges and then sum them up

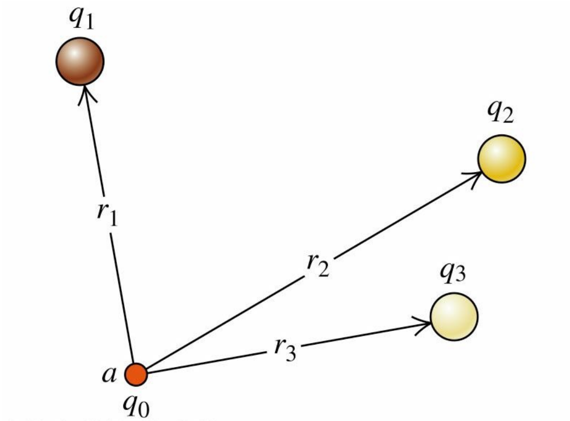
$$U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$



Electric potential energy is the amount of work done by the electric force, when taking q_0 from infinity to a.

Carrying a Stone to the Top of a Pyramid

$$U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = q_0\left(\frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}\right) = q_0 V$$



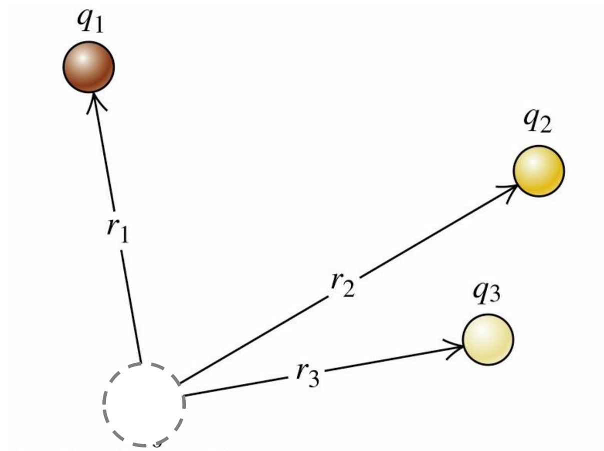
$V = \frac{U}{q_0}$ is independent of the test charge q_0, and is thus an intrinsic property of the underlying charge system $\{q_1, q_2, \ldots\}$.

- V is **electric potential**. (some textbooks use φ or ϕ for potential)
- V is the potential energy per unit charge.
- The SI unit for electric potential is called **volt** (or V for short).
- 1V = 1 volt = 1 J/C = 1 joule/coulomb

Alessandro **Volt**a
(1745 - 1827)

Electric Potential

$$U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = q_0\left(\frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}\right) = q_0 V$$



With & without q_0, the electric potential at a is always

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

For point charges

(A "hidden" convention used in the above formula is $V(\infty) = 0$)

Electric potential due to a continuous distribution of charge

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

Integral over charge distribution

Charge element

Distance from charge element to where potential is measured

Electric constant

For continuous charge distribution

Potential Difference or Voltage

The work done by the electric force in moving a test charge from point *a* to point *b* is given by

$$W_{a\to b} = U_a - U_b = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

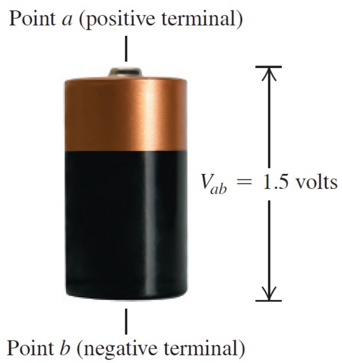
Meanwhile, we have $U_a = q_0 V_a, U_b = q_0 V_b$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

Potential Difference V_{ab}

- V_{ab} is the potential of a with respect to b.
- V_{ab} is also called the voltage.
- $V_{ab} = -V_{ba}$.
- The work $W_{a\to b}$ done by the electric force when moving a charge q_0 from a to b is

$$W_{a\to b} = q_0 V_{ab}$$



Some Remarks

$$V_{ab} = V_a - V_b = \int_a^b \vec{E} \cdot \overrightarrow{dl} = -\int_b^a \vec{E} \cdot \overrightarrow{dl}$$ (A preferred convention to rewrite the integral)

Unit of V_{ab} is volt (V); unit of length is meter (m); unit of electric field is newton/coulomb (N/C)

volt = newton/coulomb × meter or V/m = N/C

The unit of electric field is 1 N/C as well as 1 V/m.

If a charge q carries the magnitude of an electron charge, $+1.602\times10^{-19}C$, and the potential difference is $V_{ab} = 1V = 1J/C$, then the change in potential energy is

$$U_a - U_b = qV_{ab} = 1.602\times10^{-19}J \equiv 1\ eV$$

eV is short for 1 **electron volt** and it is **a unit of energy**!

Recipe for Calculating Potential

$$V_a - V_b = -\int_b^a \vec{E} \cdot \vec{dl}$$

Q: How to find V_a?

Step 1 Identify $\vec{E}(\vec{r})$

Use Coulomb's law or Gauss's law

Step 2 Choose a zero-potential reference point b, s.t. $V_b \equiv 0$

See comments below.

Step 3 Do the electric field integral from b to a

Yes, we are allowed to choose a reference point with a zero potential! The value of potential is only meaningful when that reference point is specified.

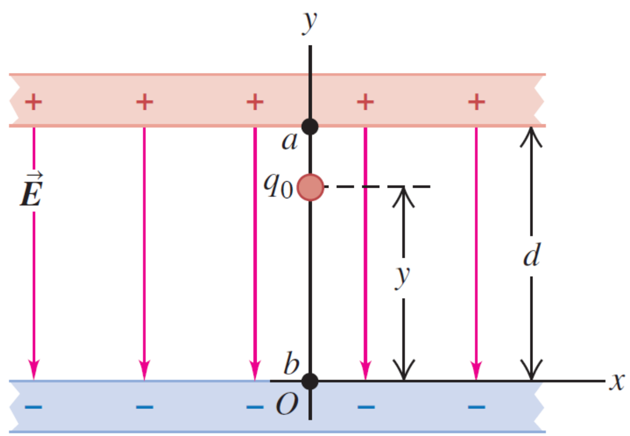
The value of potential difference is reference-point-independent!

For point charges, there exists a widely used convention $V(\infty) = 0$.

Just like your height is only meaningful when measured from the ground.

Oppositely Charged Parallel Plates (Example 23.9)

Q: Consider two infinite oppositely charged parallel plates. The surface charge density for the top & bottom plates are $+\sigma$ and $-\sigma$, respectively. Find the potential at any height y between the plates.



Step 1 Identify $\vec{E}(\vec{r})$

Using Gauss's law, it is straightforward to see that the electric field between the plates is uniform and points to $-y$ direction as

$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{y}$$

(See Example 22.8 of the textbook for details…)

Oppositely Charged Parallel Plates (Example 23.9)

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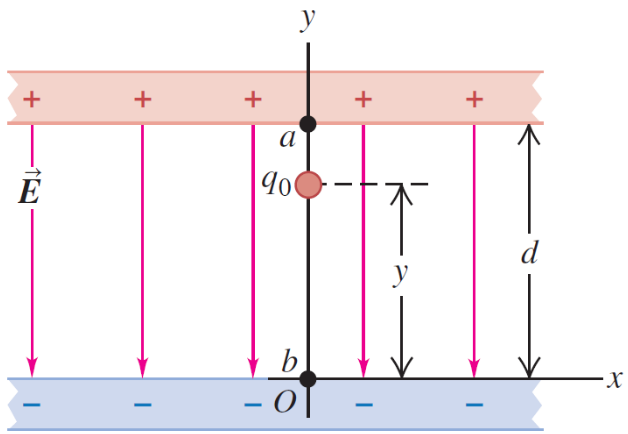


Step 2 Choose a zero-potential reference point

You can choose wherever you want!

1) Set $y_b = 0$ as the reference point (i.e. $V_b = 0$)

$$V_y = V_y - V_b = -\int_{y_b}^{y} \vec{E} \cdot \vec{dl} = \frac{\sigma}{\varepsilon_0}(y - y_b) = \frac{\sigma}{\varepsilon_0} y \Rightarrow V_a = \frac{\sigma}{\varepsilon_0} d, V_b = 0$$

2) Set $y_a = d$ as the reference point (i.e. $V_a = 0$)

$$V_y = V_y - V_a = -\int_{y_a}^{y} \vec{E} \cdot \vec{dl} = \frac{\sigma}{\varepsilon_0}(y - d) \Rightarrow V_a = 0,\ V_b = -\frac{\sigma}{\varepsilon_0} d,$$

The potential difference

$$V_{ab} = \frac{\sigma}{\varepsilon_0} d$$

Electric Field as Potential Gradient

$$V_a - V_b = -\int_b^a \vec{E} \cdot \vec{dl}$$

Inverse operation of integral is derivative!

$$\vec{E} = -\vec{\nabla} V$$

Vector Derivative (Gradient)

$$\vec{\nabla} V = \hat{\imath}\frac{dV}{dx} + \hat{\jmath}\frac{dV}{dy} + \hat{k}\frac{dV}{dz}$$

$$\vec{E} = -(\hat{\imath}\frac{dV}{dx} + \hat{\jmath}\frac{dV}{dy} + \hat{k}\frac{dV}{dz})$$

Electric field tells you how "steep" the potential distribution is!

Q: The electric potential in a region of space is given by $V(x) = 3x^2 - x^3$. The x-component of the electric field E_x at $x = 2$ is

✓ *A.* $E_x = 0$

B. $E_x > 0$

C. $E_x < 0$

$$\vec{E} = -\vec{\nabla} V = -\hat{\imath}\frac{dV}{dx}$$

$$E_x = -\frac{dV}{dx} = 6x - 3x^2 = 6\times 2 - 3\times 2^2 = 0$$

Equipotential Surface

- It is possible to move a test charge from one point to another without having any net work done on the charge
- This occurs when the starting and end points have the same potential.
- It is possible to map out such points and a given set of points at the same potential form an *equipotential surface.*

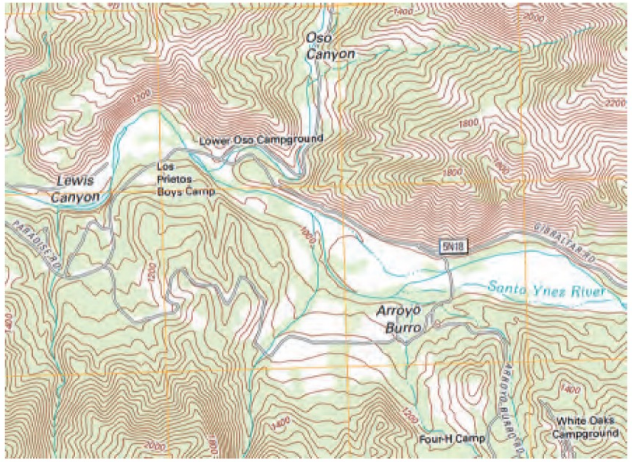
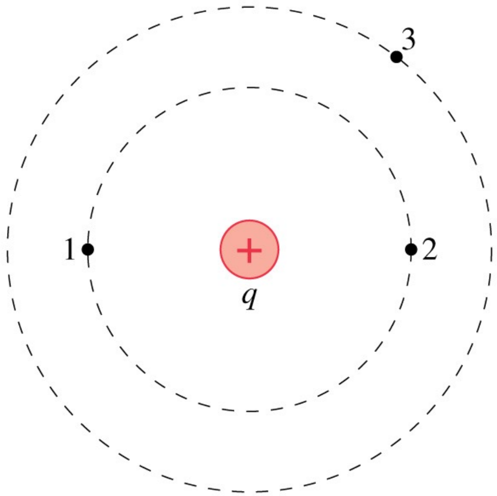
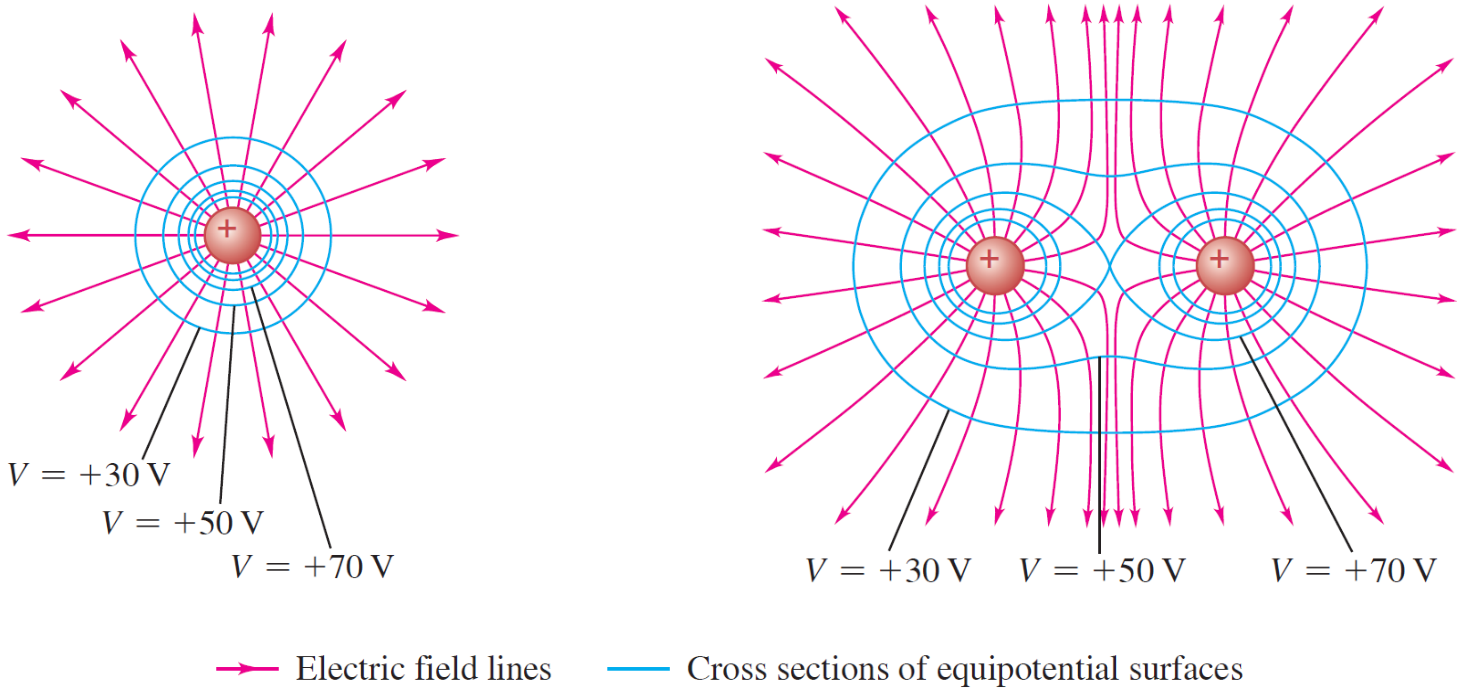


Figure **23.22** Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.



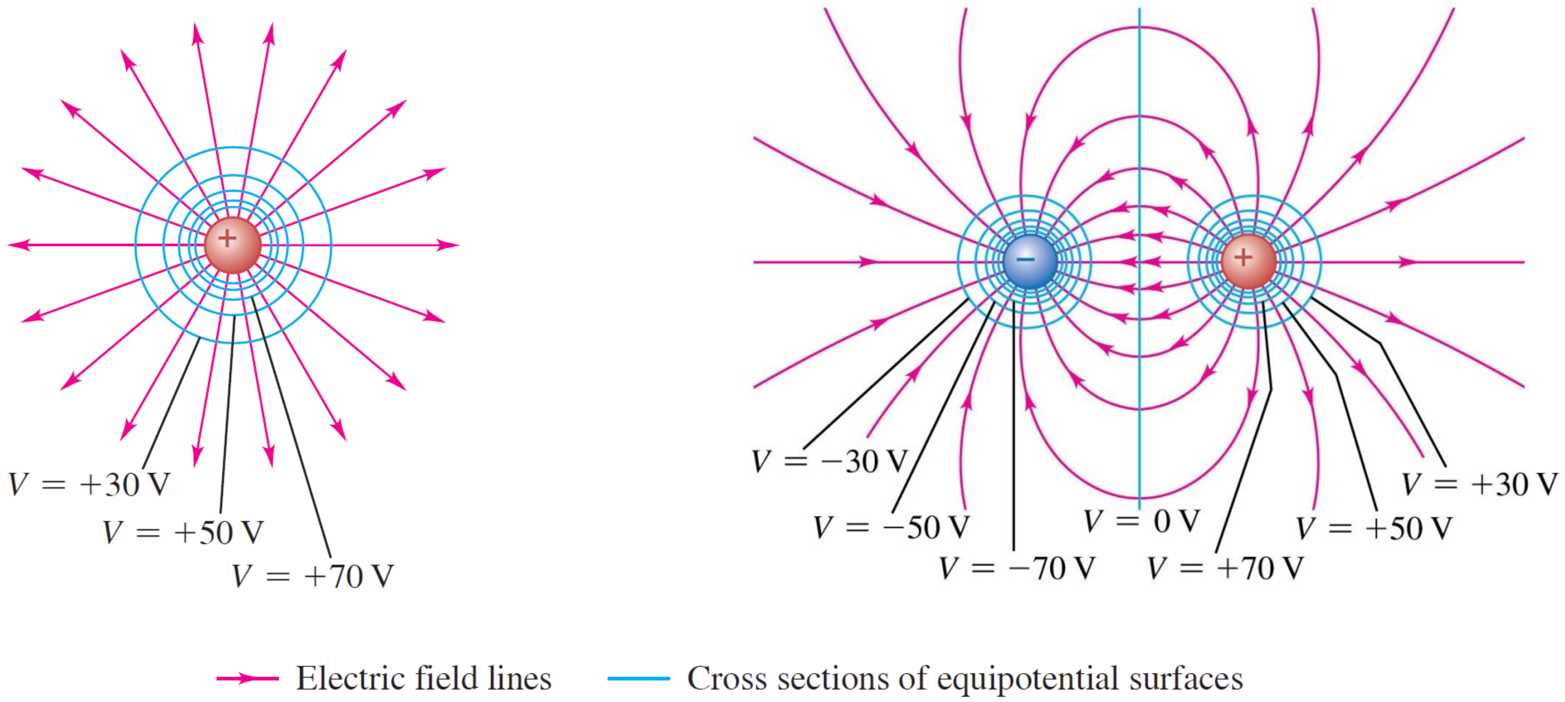
$$V_{13} = V_{23} > V_{12} = 0$$

Equipotential Surface



- The potential difference between two neighboring surfaces is the same.
- The spacing between neighboring surfaces is usually uneven.

Equipotential Surface



- Since no work is done, there is no force or field along any direction of motion within the equipotential surface.
- The electric field is **always perpendicular** to the equipotential surface. (Why?)

Potential on Conductors

In the electrostatic (no movement of charges in the conductor) limit, the electric field within the material of the conductor will everywhere be zero.

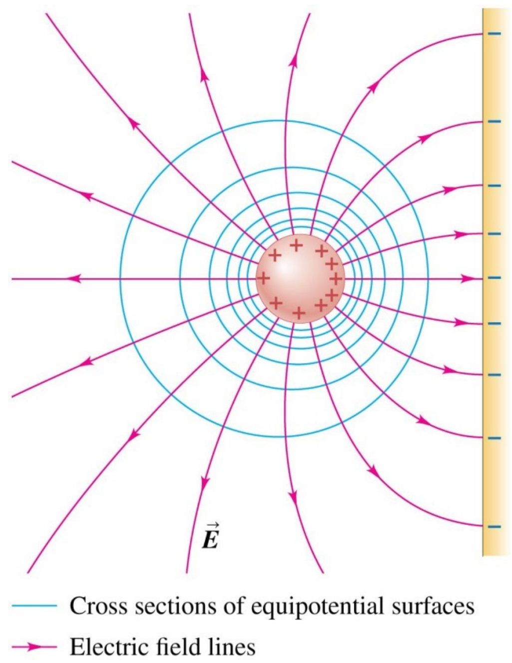
No electric field means no work can be done.

1) a conductor is an equipotential body/object.

2) The surface of a conductor is, by definition, an equipotential surface.

3) Only perpendicular electric field on the conductor surface!

(The electric field is always perpendicular to the equipotential surface)



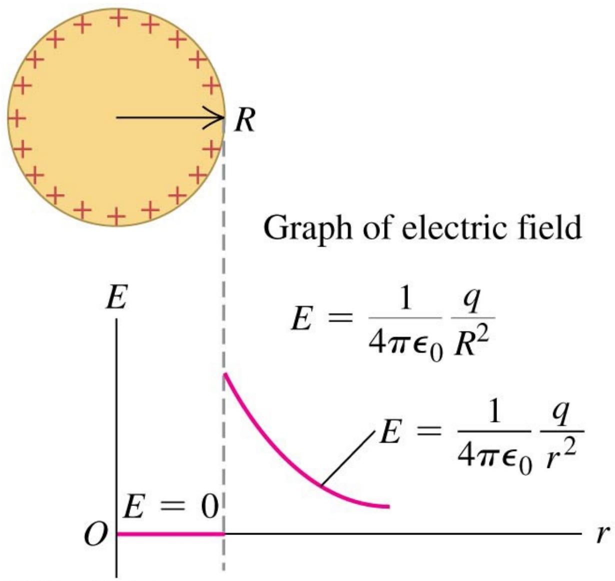
Potential on Conductors

1) The electric field inside a conductor is everywhere zero in the electrostatic limit.

2) a conductor is an equipotential body/object.

3) The surface of a conductor is, by definition, an equipotential surface.

4) Only perpendicular electric field on the conductor surface!



A conducting sphere carries charge q