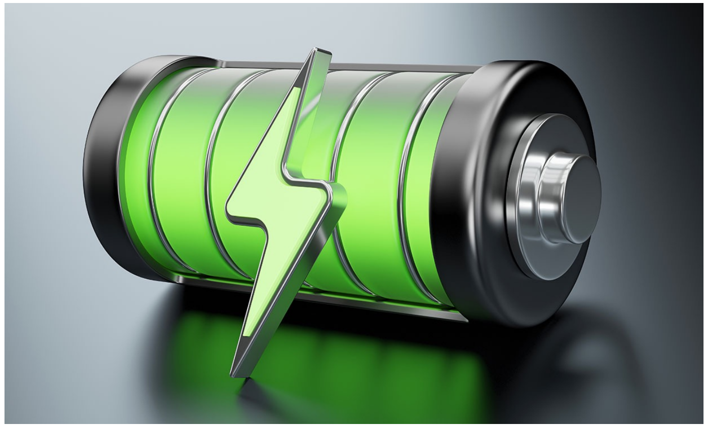
ELECTRICITY AND MAGNETISM (PHYS 231)

Lecture 6: Electric Potential energy

Sep 9, 2024

Homework #2 Due 11 pm Monday

“Volt Tackle” of Pikachu



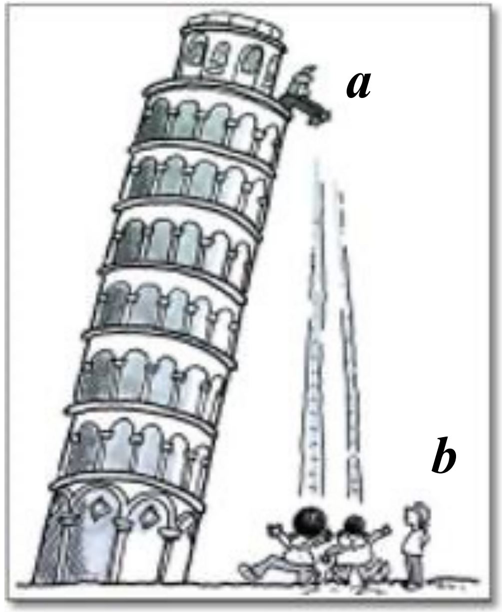
Energy, Work, & Potential Energy

- Two kinds of energy: **Kinetic** Energy & **Potential** Energy.

$$K = \frac{1}{2}mv^2 \qquad U = U(\vec{r})$$

- Total Energy (K+U) is **conserved**.
- Energy is transferrable.
- **Work** is the energy difference or the amount of the transferred energy.
- When a force, F, acts on a particle, work is done on the particle in moving from point *a* to point *b*.

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$$



$K\uparrow, U\downarrow$

Energy, Work, & Potential Energy

- If the force is **conservative**, then the work done can be expressed in terms of *a change in potential energy.*

$$W_{a\to b} = U(\vec{r}_a) - U(\vec{r}_b) = -\Delta U$$

$$K(v_a) + U(\vec{r}_a) = K(v_b) + U(\vec{r}_b)$$

A conservative force is a force with the property that the total work done in moving a particle between two points is **independent of the path taken**.

Gravitational Force is a conservative force.

$$\vec{F}_{A\to B} = -G\frac{m_A m_B}{r^2}\hat{r}_{AB}$$

Electric Force is a conservative force.

$$\vec{F}_{1\to 2} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

If we can define a potential energy for gravity, we can also doe the same for the electric force.

"Electric Potential Energy"

A Point Charge in a Uniform Electric Field

Electric force on charge is

$$\vec{F} = q_0\vec{E} = -q_0E\hat{y}$$

Work is done on the charge by electric field

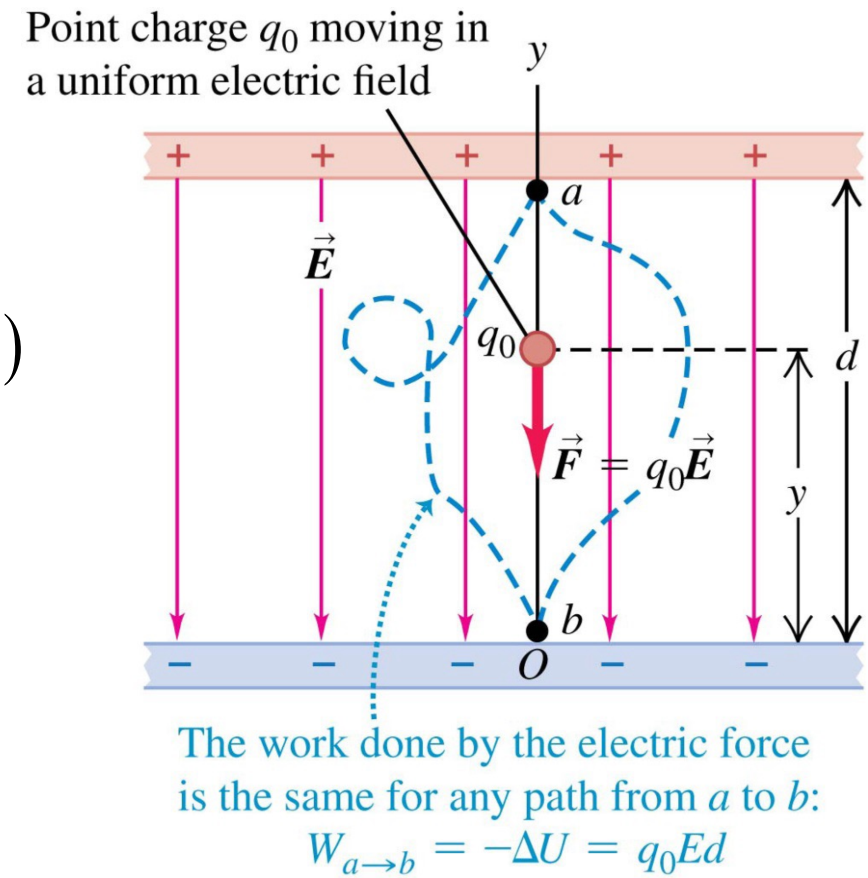
$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = -q_0E\int_a^b \hat{y}\cdot d\vec{l} = -q_0E\left(y_b - y_a\right)$$

We define the electric potential energy $U(\vec{r})$ as a function of $\vec{r}$, s.t. it satisfies

$$W_{a\to b} = U(\vec{r}_a) - U(\vec{r}_b) = -\Delta U$$

Electric potential energy @ $\vec{r}_a$

Potential energy difference from $\vec{r}_b$ to $\vec{r}_a$



Electric force is conservative!

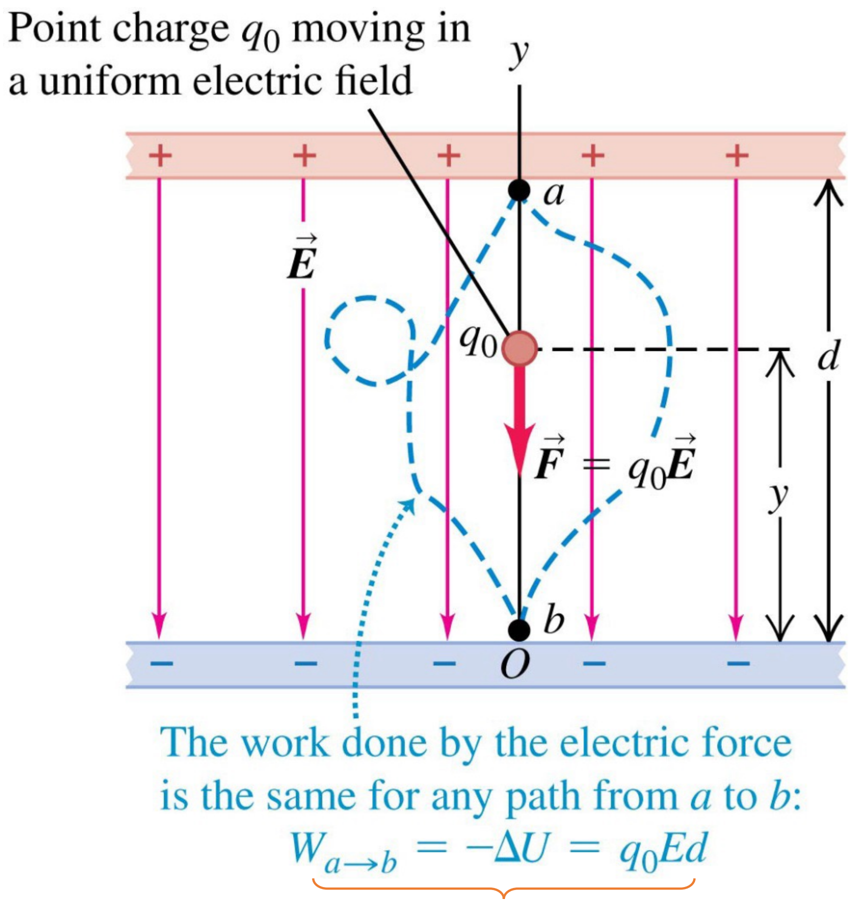
A Point Charge in a Uniform Electric Field

$$W_{a\to b} = U(\vec{r}_a) - U(\vec{r}_b) = -\Delta U$$
$$= q_0 E y_a - q_0 E y_b$$

Note that $E > 0, y_a > y_b$,

1) If $q_0 > 0$, we have $\Delta U < 0$. (lose U)
2) If $q_0 < 0$, we have $\Delta U > 0$. (gain U)

A positive (negative) test charge will lose (gain) electric potential energy if it moves *along* the field line.



Electric force is conservative!

A Point Charge in a Uniform Electric Field

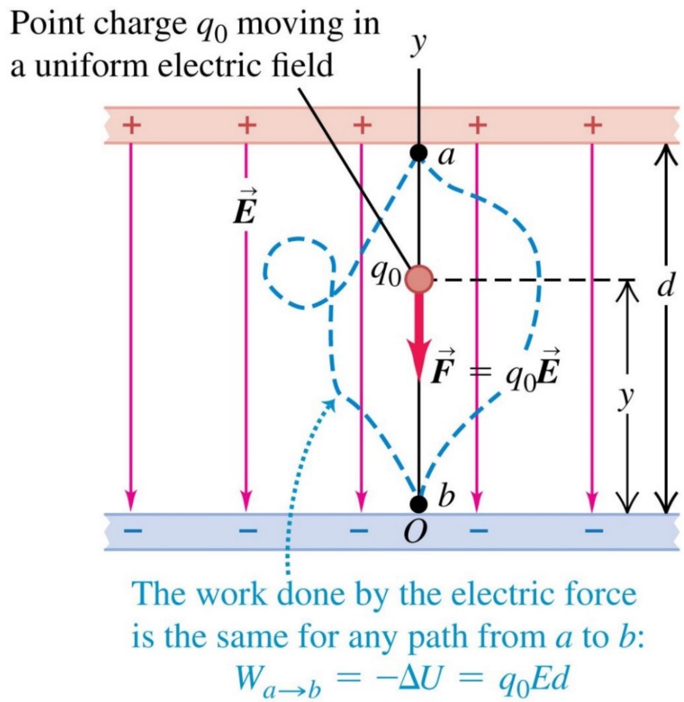
If no other force in play

$$K(v_a)+U(\vec{r}_a)=K(v_b)+U(\vec{r}_b)$$ (Energy conservation)

$$U(\vec{r}_a)-U(\vec{r}_b)=K(v_b)-K(v_a)=\frac{1}{2}m{v^2}_b-\frac{1}{2}m{v^2}_a$$

For $q_0 > 0,\ W_{a\to b} = U_a - U_b = -\Delta U = K_b - K_a > 0$

$$U_b < U_a, K_b > K_a$$



- Electric force is doing a **positive** work ($W_{a\to b} > 0$)
- Electric potential energy **decreases**. ($U_b < U_a$)
- Kinetic energy **increases** ($K_b > K_a$).
- Energy transfer from U to K, and thus remain **conserved** as a whole.

Q: A negative point charge moves from *a* to *b* with a **non-zero** initial velocity. Which of the following is true for electric potential energy U and kinetic energy K for the charge after arriving at b?

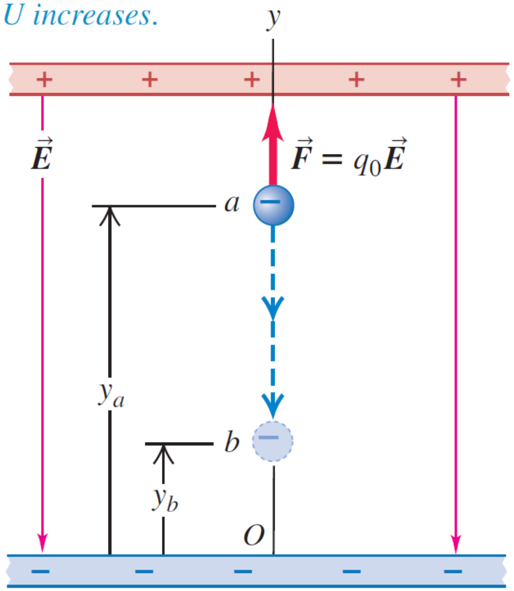
A. U increases, K increases
B. U decreases, K increases
C. U increases, K decreases
D. U decreases, K decreases
E. Not enough information

0

Q: A negative point charge moves from a to b with a **non-zero** initial velocity. Which of the following is true for electric potential energy U and kinetic energy K?

(a) Negative charge q_0 moves in the direction of $\vec{E}$:

- Field does *negative* work on charge.
- *U increases.*

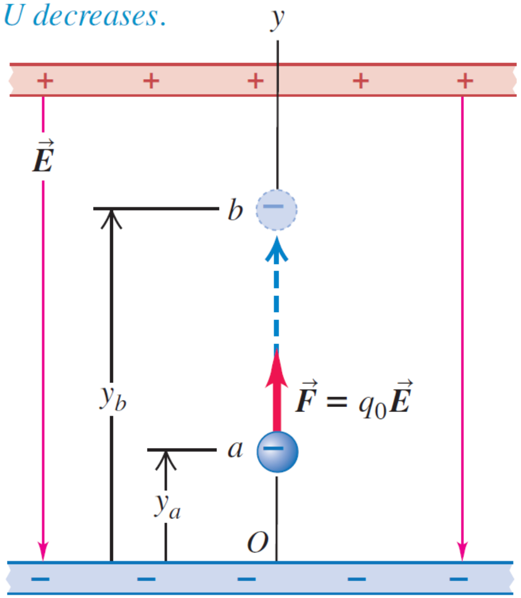


Possible ONLY with a non-zero velocity

Both scenarios are possible!

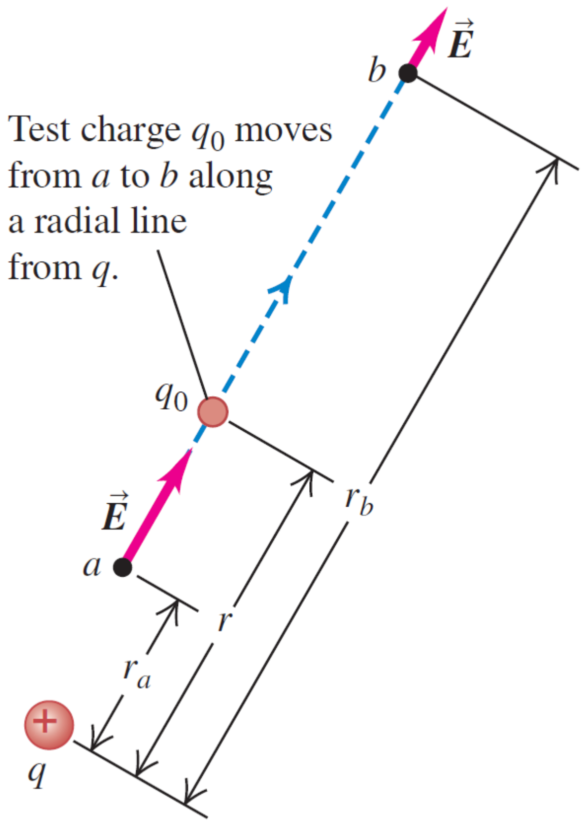
(b) Negative charge q_0 moves opposite $\vec{E}$:

- Field does *positive* work on charge.
- *U decreases.*



Possible with either a zero or a non-zero velocity

Potential Energy in a Non-uniform Electric Field



Q: Calculate the work done by the electric force by moving q_0 from $\vec{r}_a$ to $\vec{r}_b$.

Step 1 Calculate the Coulomb force.

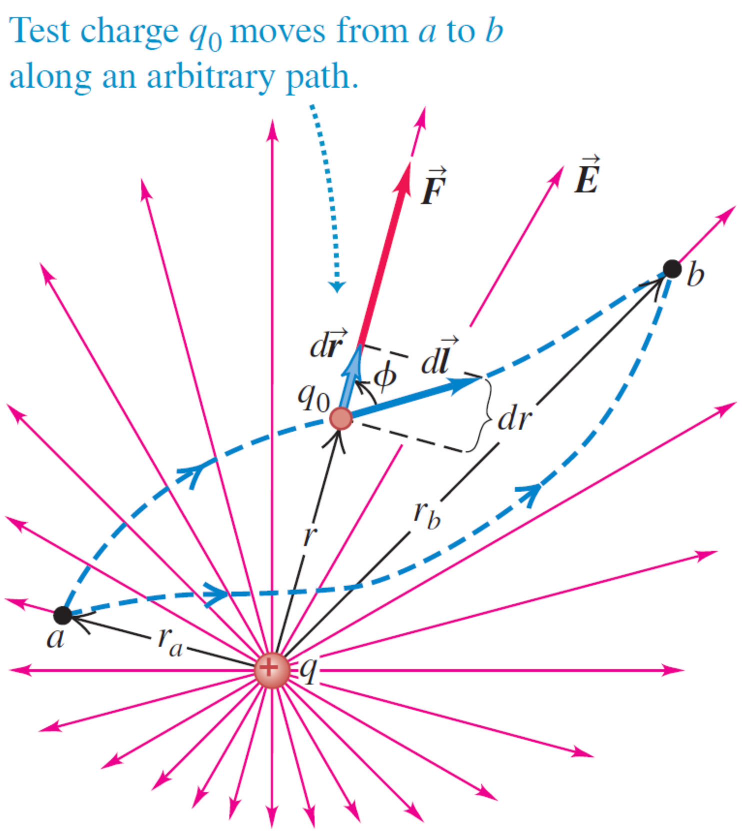
$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} = q_0\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = q_0E(r)$$

Step 2 Work is an integration of the force over the path.

$$W_{a\to b} = \int_{r_a}^{r_b} F_r dr = q_0\int_{r_a}^{r_b} E(r)dr = q_0\int_{r_a}^{r_b}\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}dr$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr = \frac{qq_0}{4\pi\varepsilon_0}\left.\frac{-1}{r}\right|_{r_a}^{r_b}$$

Potential Energy in a Non-uniform Electric Field



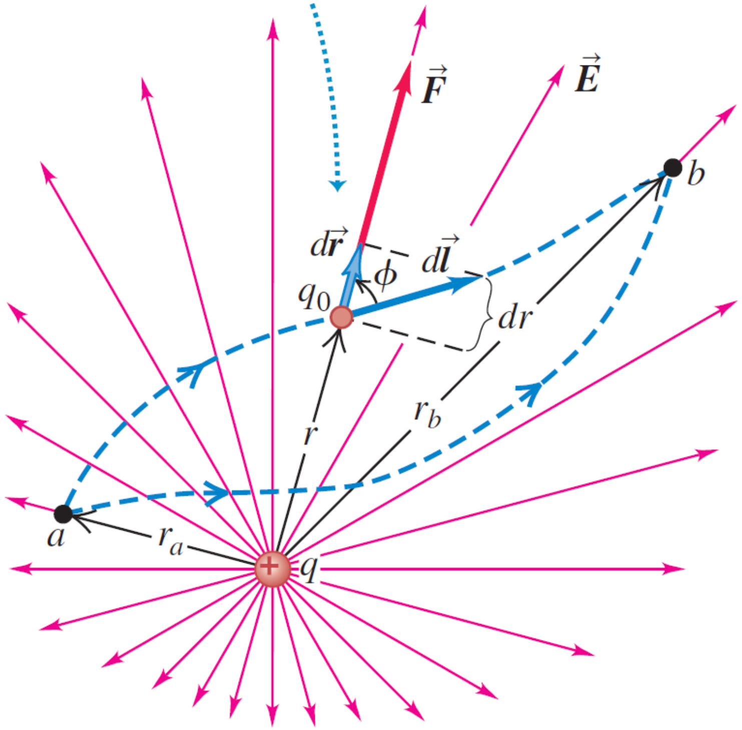
Q: Calculate the work done by the electric force by moving q_0 from $\vec{r}_a$ to $\vec{r}_b$.

$$U(\vec{r}_a) - U(\vec{r}_b) = W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

In the above calculation, we have assumed $\vec{r}_a$ & $\vec{r}_b$ are along the SAME radial direction.

What about a general displacement where a & b do NOT lie on the same radial line?

Potential Energy in a Non-uniform Electric Field



Q: Calculate the work done by the electric force by moving q_0 from $\vec{r}_a$ to $\vec{r}_b$.

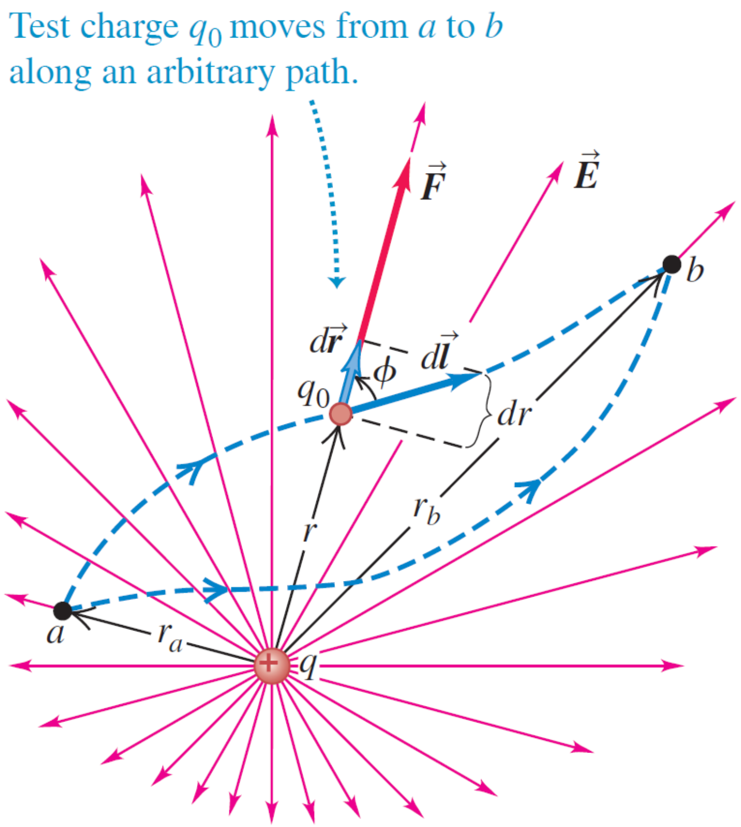
$$U(\vec{r}_a) - U(\vec{r}_b) = W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

The above result holds even when a & b do NOT lie on the same radial line!

On Page 750 of the textbook, a mathematical proof is provided.

In the following, I will provide a “physical” proof.

Potential Energy in a Non-uniform Electric Field



$$U(\vec{r}_a) - U(\vec{r}_b) = W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Moving from a to b
= Moving from a to ∞ + Moving from ∞ back to b

(Thanks to the conservative force…)

An arbitrary point and "Infinity" always lie along the same radial direction.

$$U(\vec{r}_a) - U(\infty) = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{\infty}\right) = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_a}$$

$$U(\vec{r}_b) - U(\infty) = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{\infty}\right) = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_b}$$

Potential Energy in a Non-uniform Electric Field

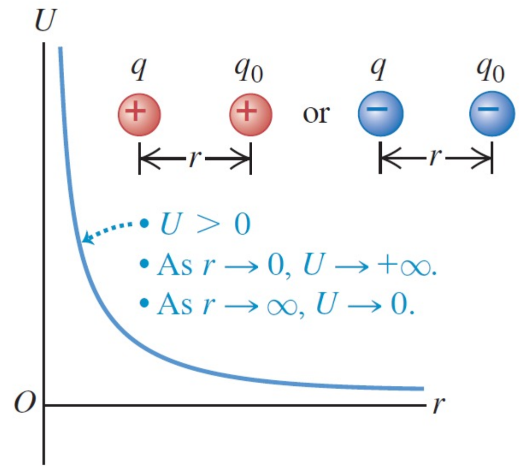
$$U(\vec{r}_a) - U(\infty) = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{\infty}\right) = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_a}$$

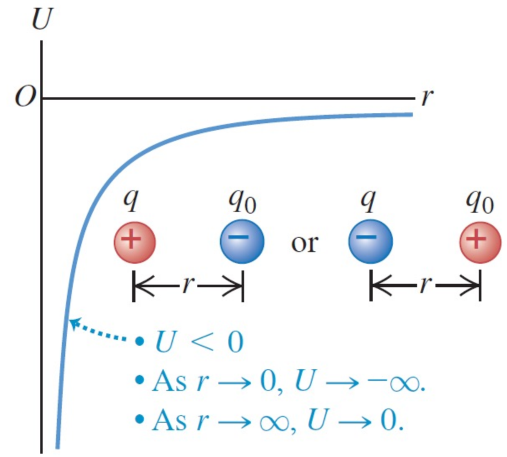
$$U(\vec{r}_b) - U(\infty) = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{\infty}\right) = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_b}$$

Why don't we define $U(\infty) = 0$ as a reference?

$$U(\vec{r}) = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r}$$

Definition of electric potential energy between two point charges.





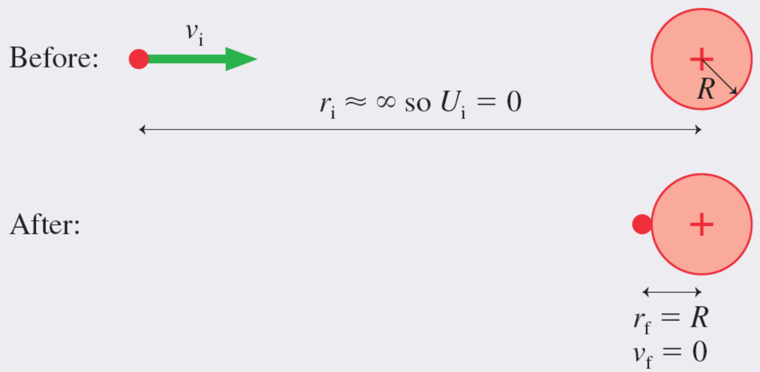
Example #1

EXAMPLE 29.2 Approaching a charged sphere

A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to +100 nC. What initial speed must the proton have to just reach the surface of the glass?

VISUALIZE **FIGURE 29.12** shows the before-and-after pictorial representation. To "just reach" the glass sphere means that the proton comes to rest, $v_f = 0$, as it reaches $r_f = 0.50$ mm, the *radius* of the sphere.

FIGURE 29.12 A proton approaching a glass sphere.



How about kinetic energy?

$$\vec{v}_f = \vec{v}_i + \int_{\infty,t=0}^{r_f,t_f} \vec{a}dt \geq 0$$

$$\int_{\infty}^{r_f} \vec{a}dt = \int_{\infty}^{r_f} \frac{\vec{F}}{m}dt = \frac{e}{m}\int_{\infty}^{r_f} \vec{E}(\vec{r})dt$$

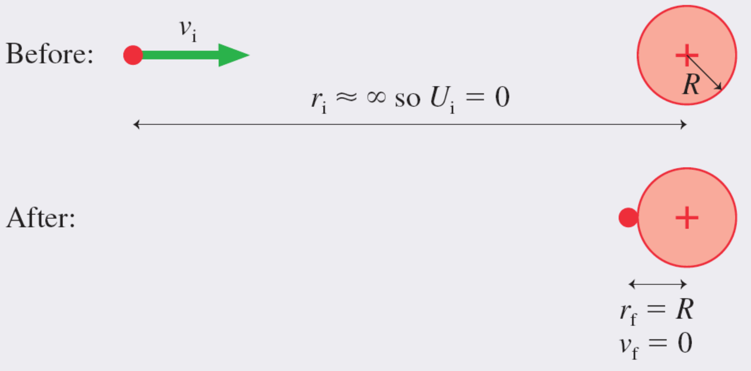
Example #1

MODEL Energy is conserved. The glass sphere can be treated as a charged particle, so the potential energy is that of two point charges. The proton starts "far away," which we interpret as sufficiently far to make $U_i \approx 0$.

$$K(v_a) + U(\vec{r}_a) = K(v_b) + U(\vec{r}_b)$$

$$U(\vec{r}_a) - U(\vec{r}_b) = K(v_b) - K(v_a) = \frac{1}{2}mv^2{}_b - \frac{1}{2}mv^2{}_a$$

FIGURE 29.12 A proton approaching a glass sphere.



How about kinetic energy?

$$\vec{v}_f = \vec{v}_i + \int_{\infty, t=0}^{r_f, t_f} \vec{a}dt \geq 0$$

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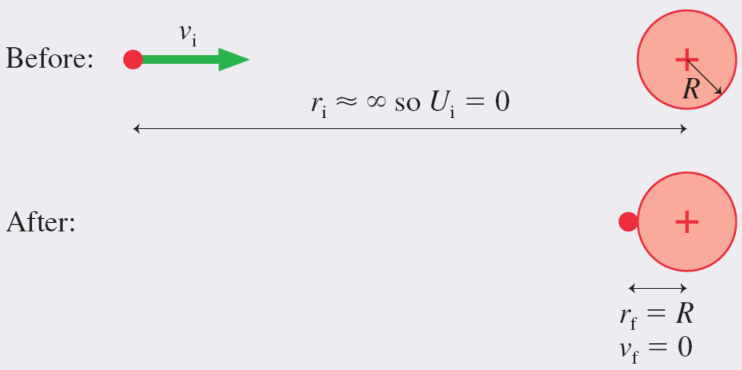
Example #1

MODEL Energy is conserved. The glass sphere can be treated as a charged particle, so the potential energy is that of two point charges. The proton starts "far away," which we interpret as sufficiently far to make $U_i \approx 0$.

SOLVE Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + \frac{Kq_p q_{sphere}}{r_f} = \frac{1}{2}mv_i^2 + 0$$

FIGURE 29.12 A proton approaching a glass sphere.



How about kinetic energy?

$$\vec{v}_f = \vec{v}_i + \int_{\infty,t=0}^{r_f,t_f} \vec{a}dt \geq 0$$

$$\int_{\infty}^{r_f} \vec{a}dt = \int_{\infty}^{r_f} \frac{\vec{F}}{m}dt = \frac{e}{m}\int_{\infty}^{r_f} \vec{E}(\vec{r})dt$$

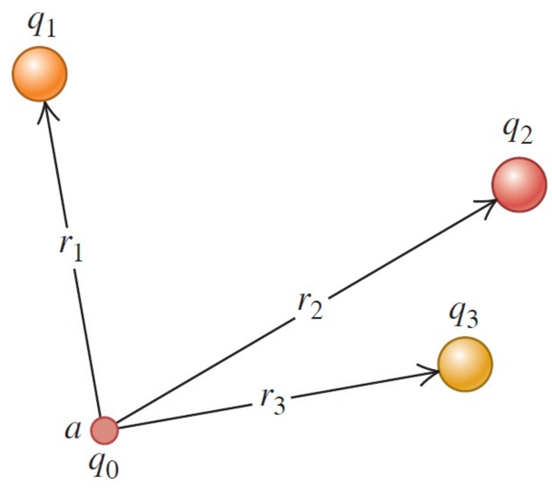
Multiple Point Charges: Two Different Potential Energies

The **potential energy associated with a test charge q_0** due to the electric field generated by other charges $q_1, q_2, q_3, \ldots$

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric constant

Distances from q_0 to $q_1, q_2, q_3, \ldots$



Meanwhile, $q_1, q_2, q_3, \ldots$ also have their own electric potential energies. We can thus define the concept of **total potential energy of a point-charge system** as

$$U_{tot} = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

Just Like Building a Pyramid

The potential energy associated with a test charge q_0

The effort of carrying a stone q_0 to the top of pyramid formed by q_1, q_2, q_3...

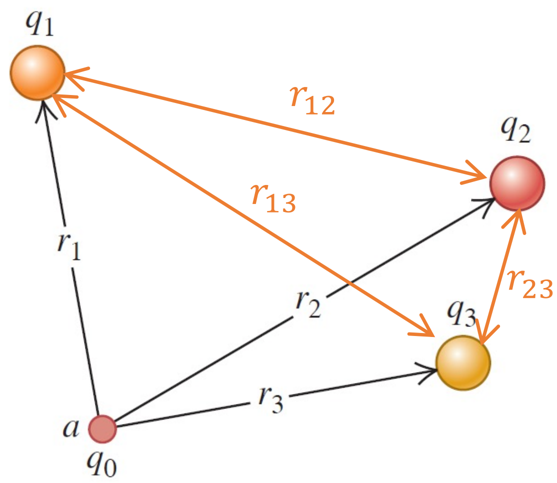
The total potential energy

Building a pyramid from the ground…

Multiple Point Charges

The potential energy for q_0 with respect to all three other charges is

$$U = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_0 q_i}{r_i} = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$



The total potential energy for all **four** charges is

$$U_{tot} = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$
$$= \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{01}} + \frac{q_2}{r_{02}} + \frac{q_3}{r_{03}}\right) + \frac{q_1}{4\pi\varepsilon_0}\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{23}}\right) + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}$$

(We have denoted $r_{0i} \equiv r_i$ for short)

Be very careful about ***which potential energy*** the question is asking about!

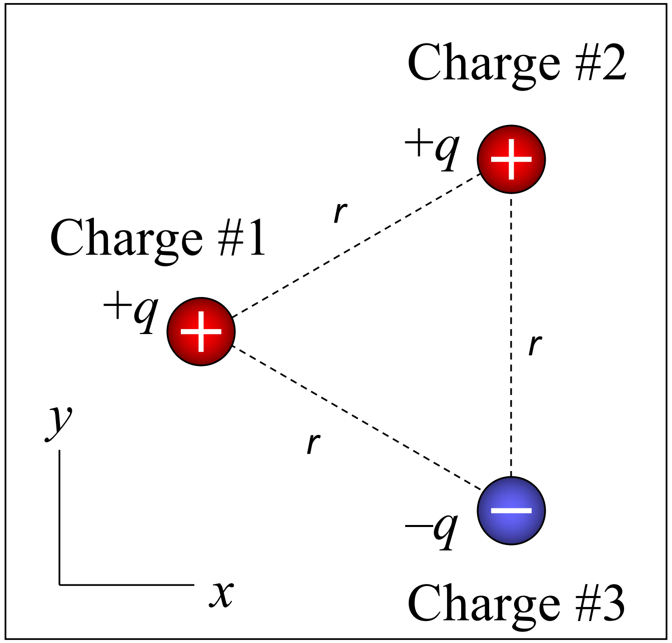
Q3: If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of charge #1 is

A. Positive

B. Negative

✔ C. Zero

D. Not enough information



Q4: If the three point charges shown here lie at the vertices of an equilateral triangle, the total potential energy of all three charges is

A. Positive

✓ B. Negative

C. Zero

D. Not enough information