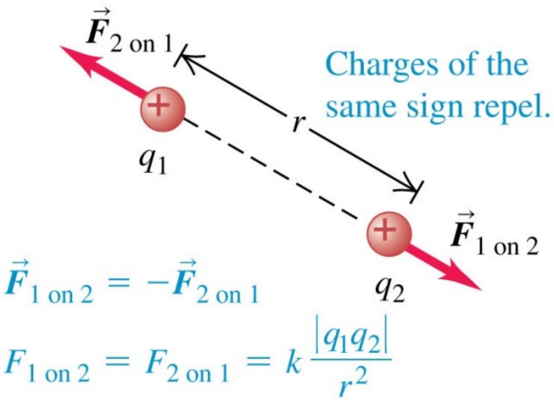
ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 4: Electric Flux and Gauss's Law

Aug 28, 2024

Homework #1 Due 11 pm Today !

Review on Coulomb's Law



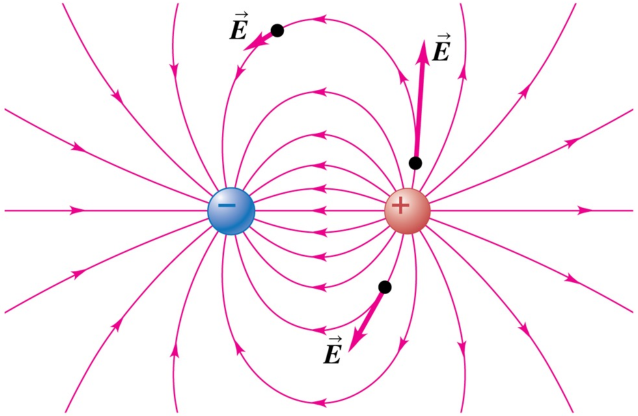
$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Coulomb's Law

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

Electric Field

Field Line & Dipole



Value of k	Units
$8.987\ 551\ 7923(14) \times 10^9$	$N{\cdot}m^2/C^2$

Spoiler for This Lecture

Charles-Augustin de Coulomb
(1736 - 1806)

Johann Carl Friedrich Gauss
(1777 - 1855)

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Why Gauss's Law?

Gauss's law is a law relating the distribution of electric charge to the resulting electric field.

(sounds like Coulomb's law, right?)

Theoretically

Gauss's law is **more fundamental** than Coulomb's law (that's why it's one of the Four!)

1) One can derive Coulomb's law from Gauss's law.

2) Coulomb's law ONLY works for stationary charges.

Practically

Gauss's law can significantly simplify ***E***-field calculation, when certain **symmetry** exists.

Gauss's Law

*The net **electric flux** through any **closed surface** is equal to $1/\epsilon_0$ times the net electric charge within that closed surface.*

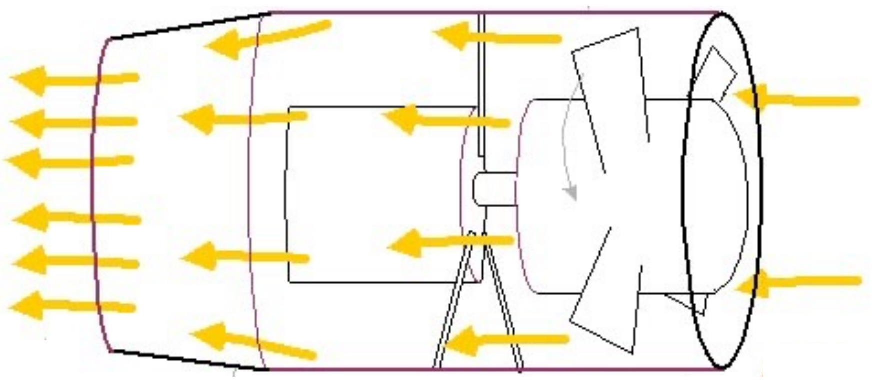
Flux

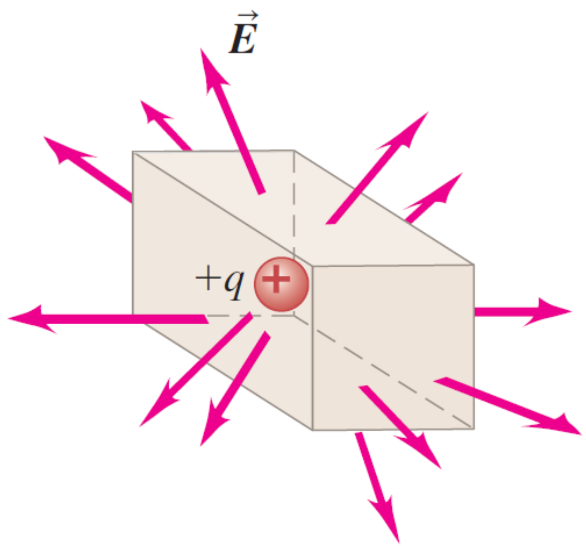
In plain language

Flux = The amount of something that passes through some surface.

In vector calculus

Flux = The surface integral of the perpendicular component of a vector field over a surface.





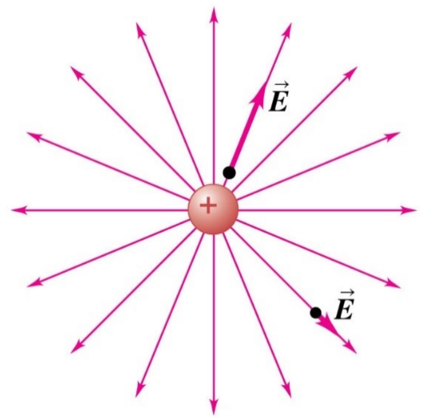
Electric Flux

Consider a uniform electric field $\vec{E}$ and a flat surface A

$$\Phi_E = \vec{E} \cdot A\hat{n} \qquad \text{(Electric flux going through } A\text{)}$$

- $\hat{n}$ is the unit vector of the surface normal.
- **Φ_E can be viewed as the number of field lines crossing through A.**

Electric Field Lines:
Magnitude: Line Density (space between lines)
Direction: Tangent along the arrow



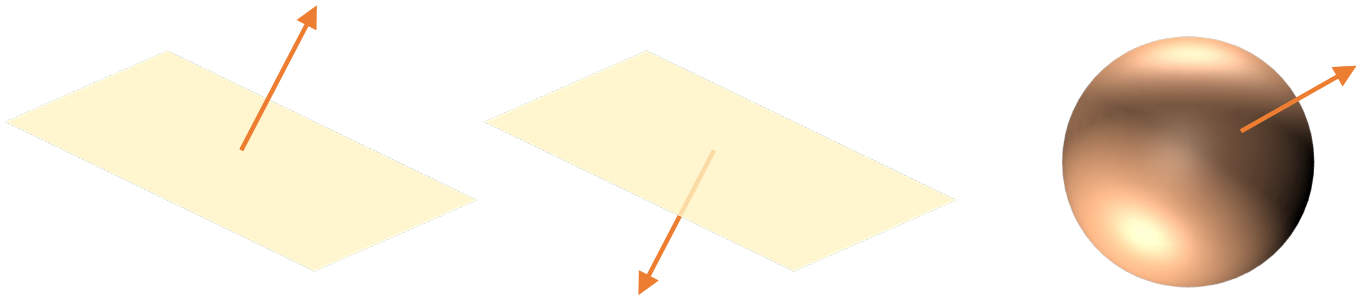
Area Vector

$$\Phi_E = \vec{E} \cdot A\,\hat{n} = \vec{E} \cdot \vec{A}$$

Magnitude of $\vec{A}$ = Area of the surface

Direction of $\vec{A}$ = Normal to the surface

Ambiguity of $\vec{A}$?



- For an open surface, the direction of $\vec{A}$ must be specified before calculating Φ_E.
- For a closed surface (e.g. a spherical surface), we always choose $\vec{A}$ to be **pointing outward** (i.e. from inner to outer).

Electric Flux

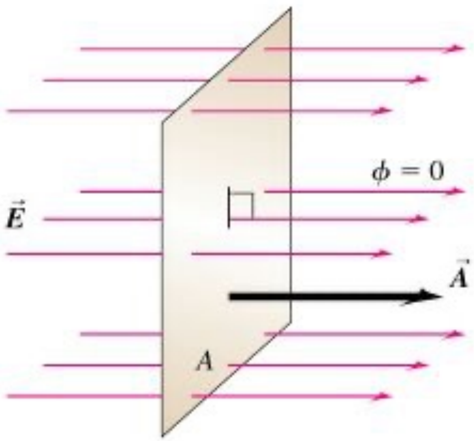
$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot A\,\hat{n} = EA\cos\phi = EA_{\perp}$$

Projected Area

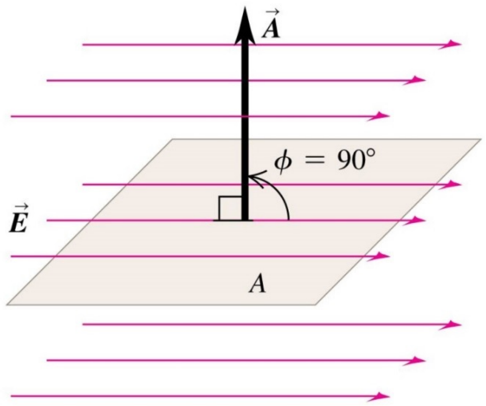
Area Vector

Unit Normal

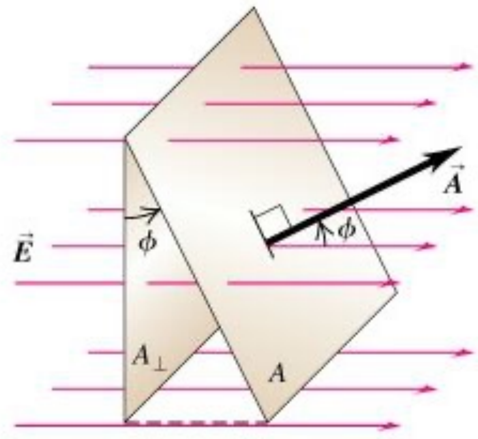
Angle between $\vec{E}$ and $\hat{n}$



$\Phi_E = EA$



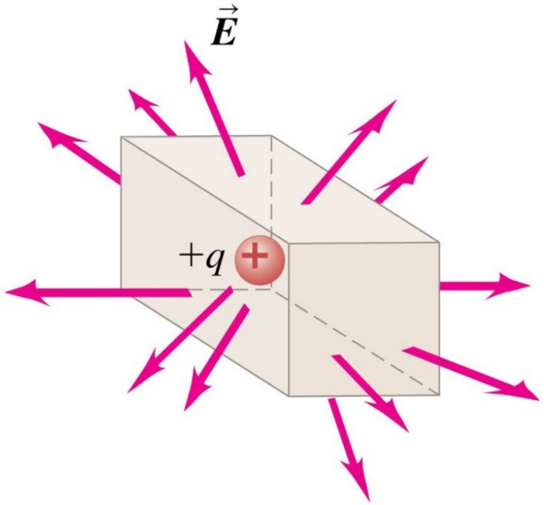
$\Phi_E = 0$



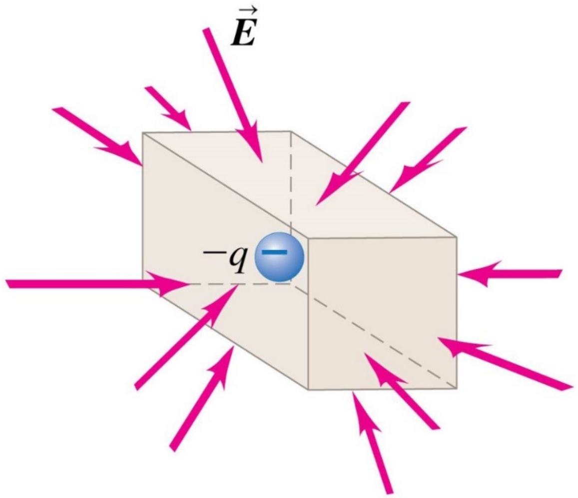
$\Phi_E = EA_{\perp} = EA\cos\phi$

Φ_E can be viewed as the number of field lines crossing through A.

Electric Flux & Enclosed Charge



$\Phi_E > 0$
(outward flux)



$\Phi_E < 0$
(inward flux)

Positive & negative charges are the "source" & "drain" of electric field.

Not quiz!

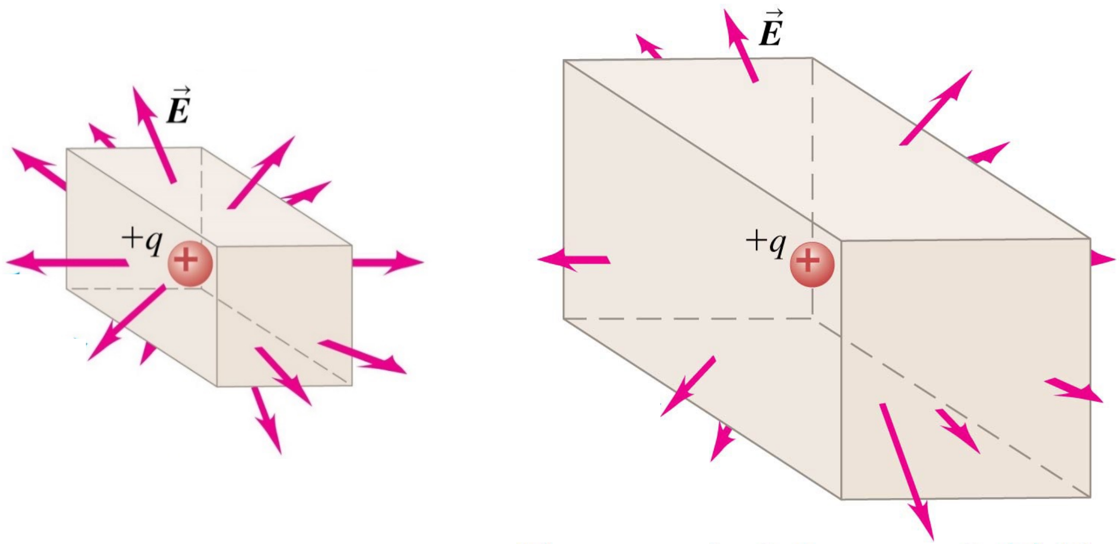
Q: What will happen to the net flux with an enlarged box?

✔ A. Same

B. Increase

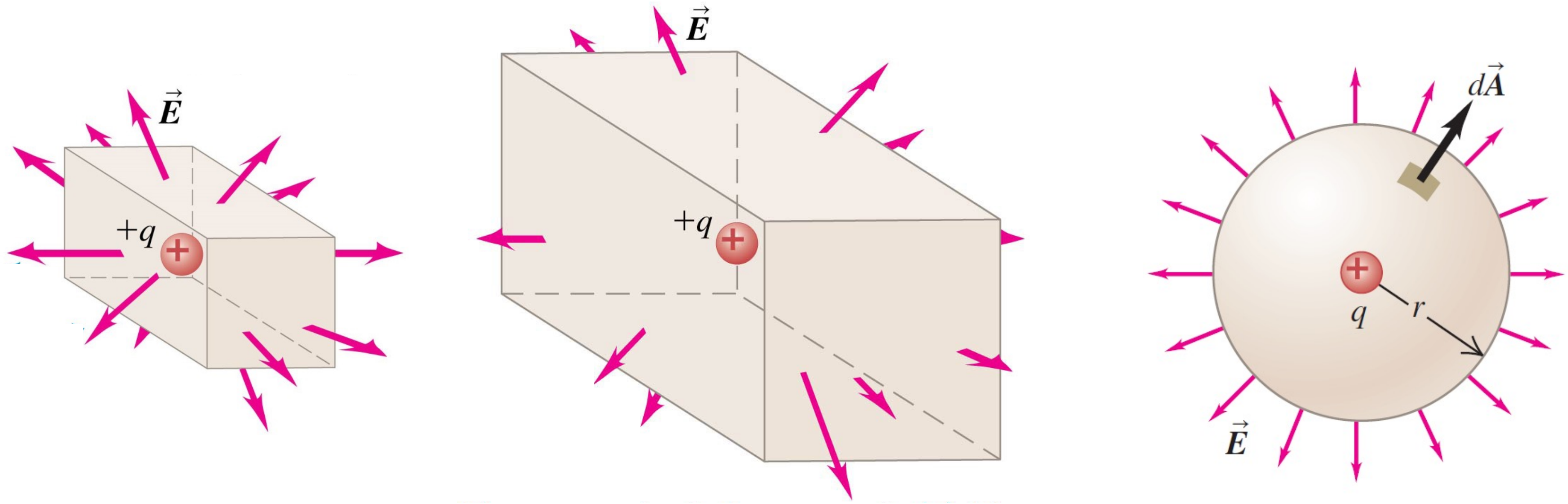
C. Decrease

D. Not Enough Information



Electric Flux & Enclosed Charge

The net flux for a closed surface does NOT depend on the size & shape of the surface.

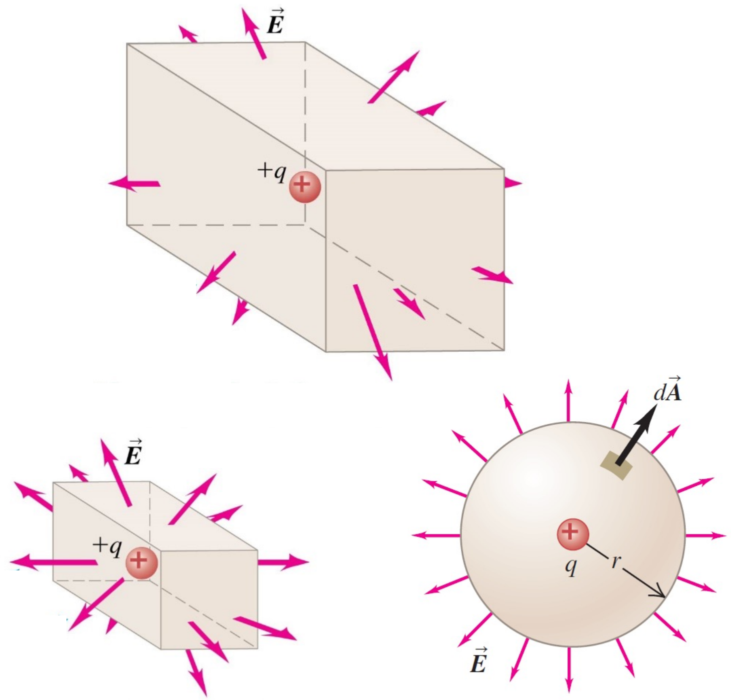


Gauss's Law in Plain Language

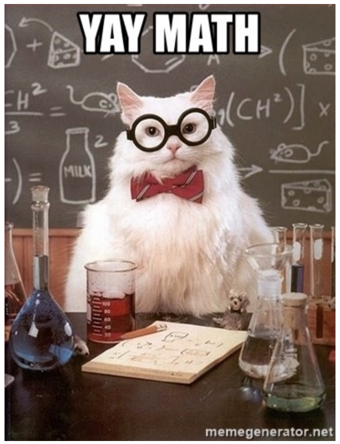
The net flux for any closed surface **ONLY** depends on the net charge enclosed.

A simple hand-waiving "proof":

1) Electric flux = # of field lines penetrating the surface.
2) # of field lines is completely determined by the charges.
3) Electric flux = charge enclosed.



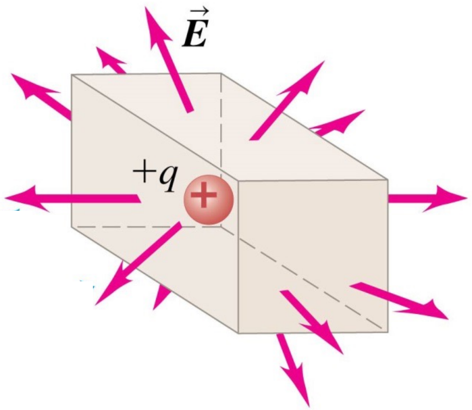
Let's now make Gauss's law a little bit mathematical!



Update the Definition of Flux

$\Phi_E = \vec{E} \cdot \vec{A}$ only works for both a uniform $\vec{E}$ and a flat surface.

This definition breaks down even for a point charge in a box…



Don't worry, we still have Calculus!

Update the Definition of Flux

Step 1

Break the surface area $\vec{A}$ into small pieces $d\vec{A}$

Step 2

Calculate the flux through each piece

$$d\Phi = \vec{E} \cdot d\vec{A}$$

Step 3

Sum them. This just becomes an area integral.

$$\Phi_E = \iint_A d\Phi = \iint_A \vec{E} \cdot d\vec{A}$$

Since each $d\vec{A}$ is really small, the electric field passing through can always be viewed as uniform up to very good approximation.

↓

Our old definition is valid for each $d\vec{A}$

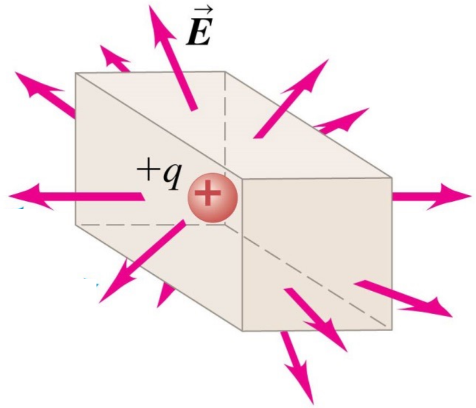
Flux of a Point Charge

For a point charge, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ ➡ $d\Phi_E = \vec{E}\cdot d\vec{A} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\cdot d\vec{A}$

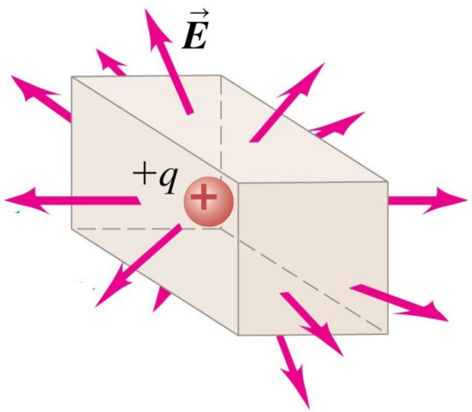
$$\Phi_E = \iint_{\text{Box}} \vec{E}\cdot d\vec{A} = \frac{q}{4\pi\varepsilon_0}\iint_{\text{Box}} \frac{1}{r^2}\hat{r}\cdot d\vec{A}$$

Surface integral over a box is doable, but difficult!

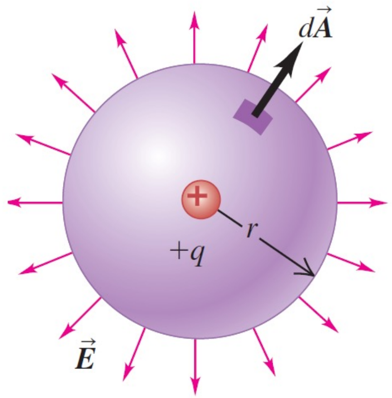
Since the flux integral does **NOT** depend on the shape of the closed surface, can we pick a "better" surface to simplify the integral?



A Symmetric Surface is a "Better" Surface



$d\Phi$ differs from place to place



$d\Phi$ is the same everywhere

$(\hat{r} = \hat{n})$

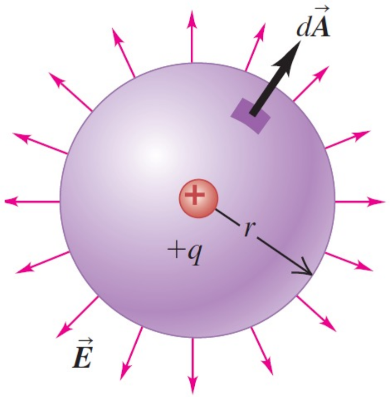
$$\Phi_E = \iint_{\text{Sphere}} \vec{E} \cdot d\vec{A} = \frac{q}{4\pi\varepsilon_0} \iint_{\text{Sphere}} \frac{1}{r^2} \hat{r} \cdot d\vec{A} = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} \oiint_S \hat{r} \cdot \hat{n} dA = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} \oiint_S dA$$

Gauss's Law

$$\Phi_E = \iint_{\text{Sphere}} \vec{E} \cdot d\vec{A} = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} \oiint_S dA$$

surface area of a sphere
(how to calculate it?)

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

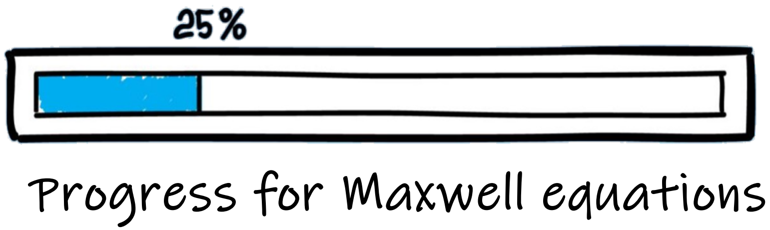


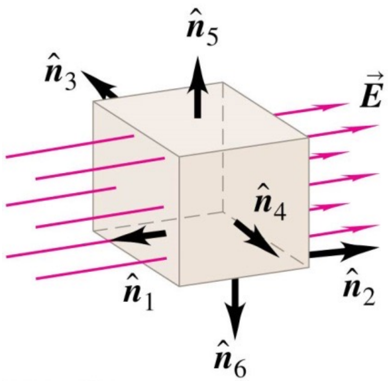
Gauss's Law

The net electric flux *through any* closed surface *is equal to* $1/\epsilon_0$ *times the net electric charge within that closed surface.*

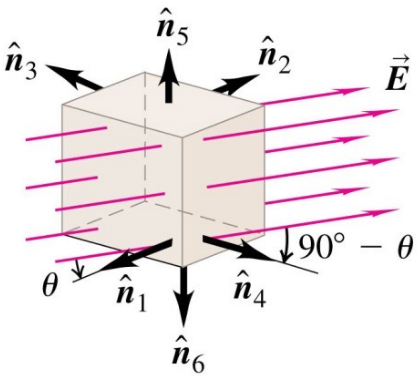
Gauss's Law

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$





$$\Phi_E = 0$$



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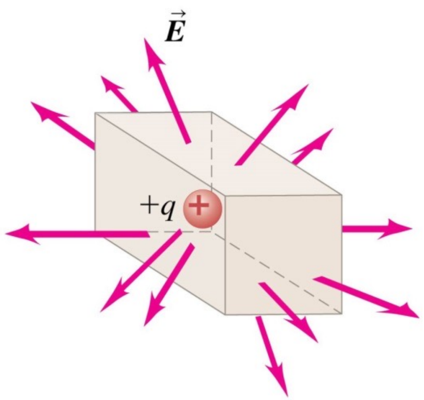
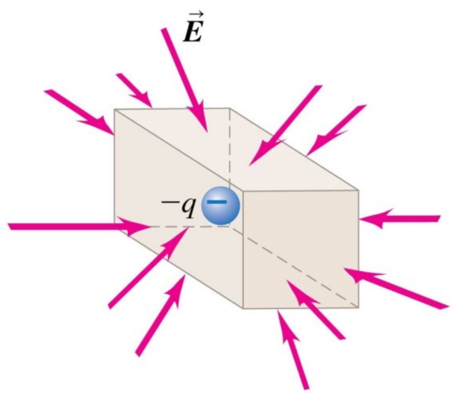


$$\Phi_E = \frac{q}{\varepsilon_0}$$



$$\Phi_E = -\frac{q}{\varepsilon_0}$$

From Gauss's Law to Coulomb's Law?

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

⇒

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Application of Gauss's Law

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

- Given an $\vec{E}$ distribution, Gauss's law can tell us the charge distribution.
- Given a charge distribution, Gauss's law can tell us the $\vec{E}$ distribution.
- Calculating $\vec{E}$ with Gauss's law **can** be much simpler than that with Coulomb's law in certain circumstances.

Coulomb or Gauss, that is THE question

Usually, but NOT always

- Coulomb: Discrete Point Charge
- Gauss: Symmetric Continuous Charge Distribution

Review on Coulomb's Law

The ionic molecule potassium bromide (KBr), made up of a positive potassium ion (K^+) of charge $+e = 1.60 \times 10^{-19}\ \mathrm{C}$ and a negative bromine ion (Br^-) of charge $-e = -1.60 \times 10^{-19}\ \mathrm{C}$, has an electric dipole moment of $3.50 \times 10^{-29}\ \mathrm{C \cdot m}$. Calculate the distance between the two ions.

At a certain point along the axis that connects the ions, the electric field due to the KBr molecule has magnitude $8.00 \times 10^{4}\ \mathrm{N/C}$. How far from the center of the molecule is this point?

Express your answer with the appropriate units.