ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 3: Electric Field and electric dipole

Aug 26, 2024

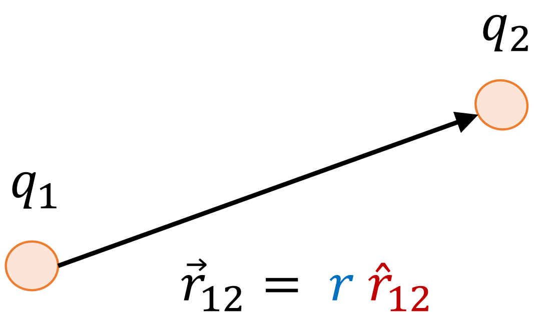
Homework Due 11 pm Aug. 28

Lab starts this week !

How Do Charges Interact?

Coulomb's Law

$$\vec{F}_{1\to 2} = k\,\frac{q_1 q_2}{r^2}\,\hat{r}_{12}$$



How do charges interact without direct contact?

Step 1

Charge q_1 modifies the properties of space around it.

Electric Field

Step 2

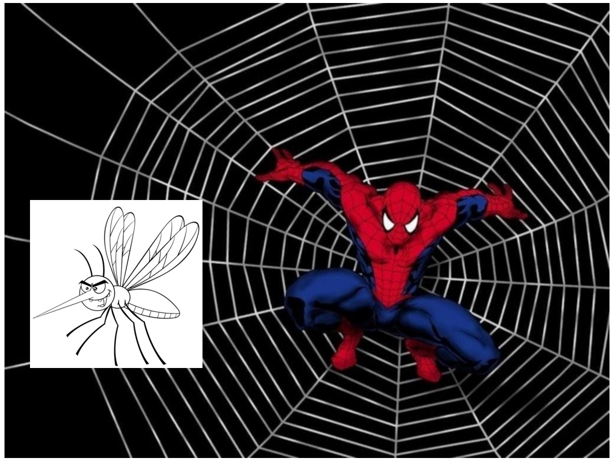
Another charge q_2 senses how the space has been modified.

Step 3

The electric response of q_2 is to experience a Coulomb force $\vec{F}_{1\to 2}$.

How Do Charges Interact?

Spiderman v.s. Mosquito



Step 1

Spiderman creates a web around him.

Step 2

Mosquito senses the web but it's too late…

Step 3

Mosquito gets trapped due to the force from the spider web.

- Electric Field is just like an invisible & infinite spider web!
- The electric force on a charged object is exerted by the electric field created by other charged objects.

What is a Field?

Mathematically

A field $f(x, t)$ is a function of space and time.

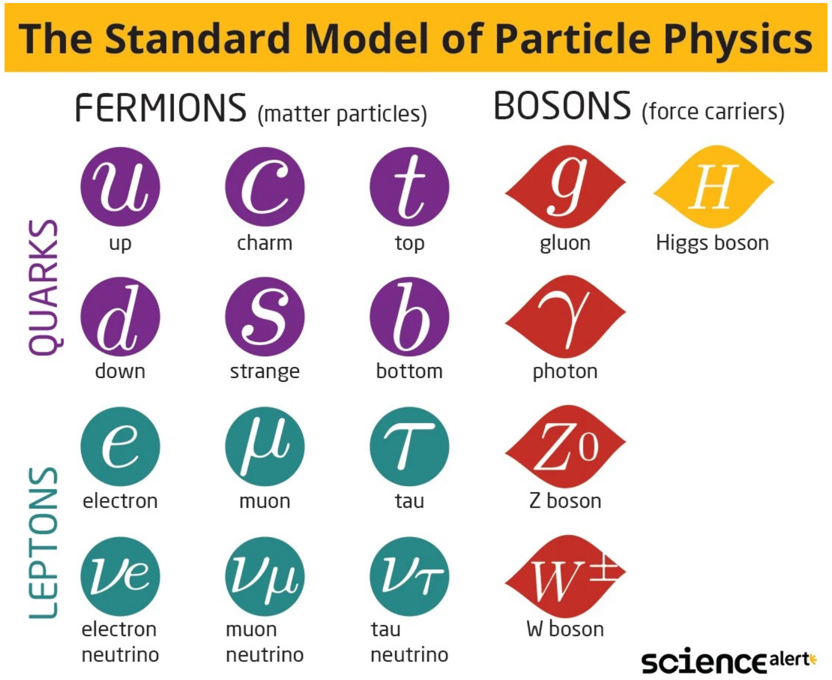
Physically

A field is a REAL thing, i.e., it carries energy, mass, momentum, obeys equation of motion, etc.

Modern Perspective from Quantum Field Theory

- Electric field (or light) is a vector field.
- The "God particle", Higgs boson, is a scalar field.
- Electron is a "spinor" field.
- Gravity is a "tensor" field.

In summary, everything is a field!



Electric Field of a Point Charge

Electric field due to a point charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

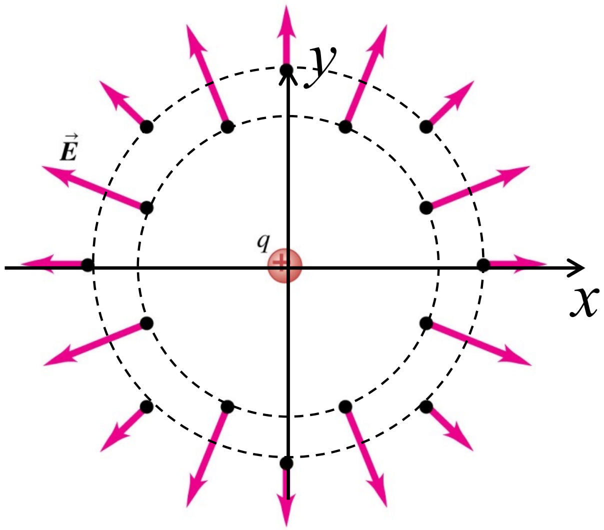
Value of point charge

Unit vector from point charge toward where field is measured

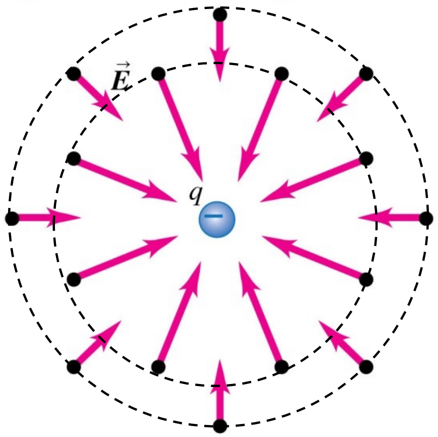
Distance from point charge to where field is measured

Electric constant

$\vec{E}$ is a vector!

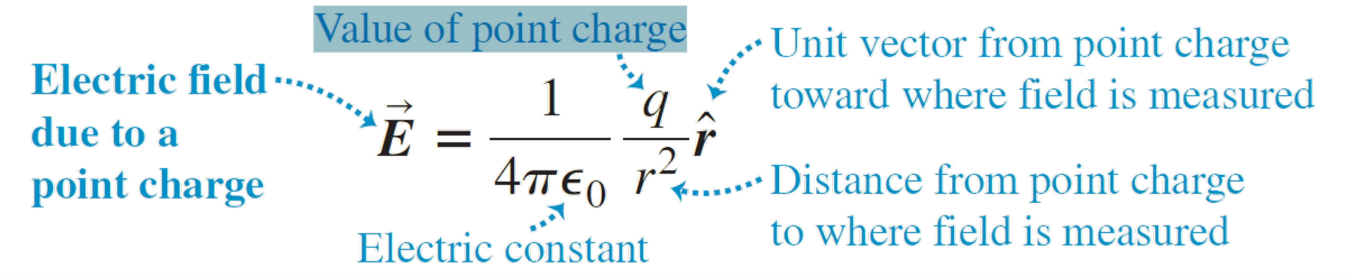


The field produced by a positive point charge points **away from** the charge.



The field produced by a negative point charge points **toward** the charge.

Electric Field of a Point Charge



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Electric field due to a point charge

Value of point charge

Unit vector from point charge toward where field is measured

Distance from point charge to where field is measured

Electric constant

Consider

1) the electric field $\vec{E}_1$ generated by a point charge q_1
2) a test charge q_2
3) The displacement between q_1 and q_2 is $\vec{r}_{12}$

$$\boxed{\vec{F}_{1\to 2} = q_2\vec{E}_1} = q_2\left(k\frac{q_1}{r^2}\hat{r}_{12}\right) = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

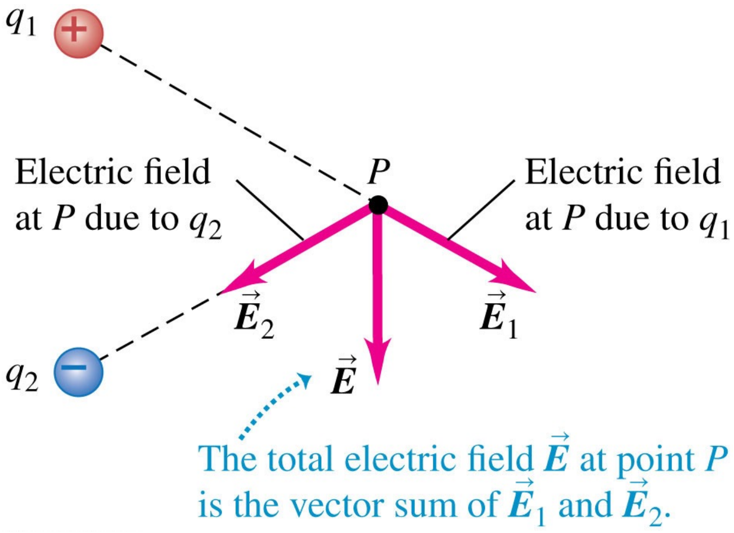
Recovers Coulomb’s law!

Electric Field Superposition

Net electric field from multiple point charges

$$\vec{E} = \sum_i \vec{E}_i = \sum_i k \frac{q_i}{r_i^2} \hat{r}_i$$

- $\vec{E}_i$ is the electric field at P generated by charge q_i.
- Electric fields add like vectors (because they are!).



Place a test charge q at P, it will experience a force $\vec{F} = q\vec{E}_p$

Calculating Electric Field

For a discrete set of point charges $q_1, q_2, q_3, \ldots$

$$\vec{E} = \sum_i \vec{E}_i = \sum_i k \frac{q_i}{r_i^2} \hat{r}_i$$

1) Calculate the electric field for each of them.

2) Sum the fields up.

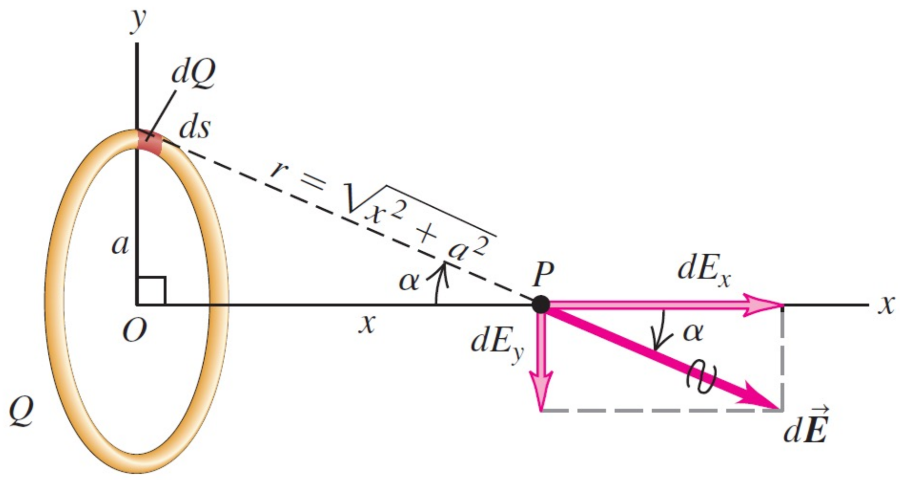
Q: What if charges are NOT point-like but are distributed continuously?

e.g. a uniformly charged stick/ring/plane/cloud…

A: We only know how to deal with point charges. If there's no point charge, we will create one!

Electric Field for a Ring of Charge

Charge Q is uniformly distributed around a conducting ring of radius a. Find the electric field at a point **P** on the ring axis at a distance x from its center.



(Example 21.9 on Page 697)

The Recipe

Step 1 Create our own point charge

Consider an infinitesimal segment ds and treat it as a point charge dubbed dQ.

Step 2 Identify the charge amount of dQ

Step 3 Calculate dE, the electric field contribution of dQ

Using the E-field formula of point charge

Step4 Sum over the contributions from all other dQ.

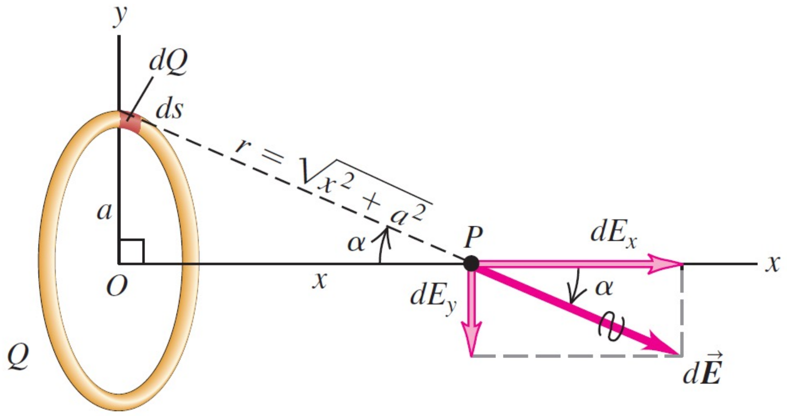
Perform an integration of dE for the entire ring.

Step 2 Identify the charge amount of dQ

Concept of Charge Density

Extremely helpful when dealing with problems with a uniform charge distribution.

- For a uniformly charged line of length L, the linear charge density is $\lambda = Q/L$
- For a uniformly charged plane of area A, the surface charge density is $\sigma = Q/A$
- For a uniformly charged space of volume V, the volume charge density is $\rho = Q/V$



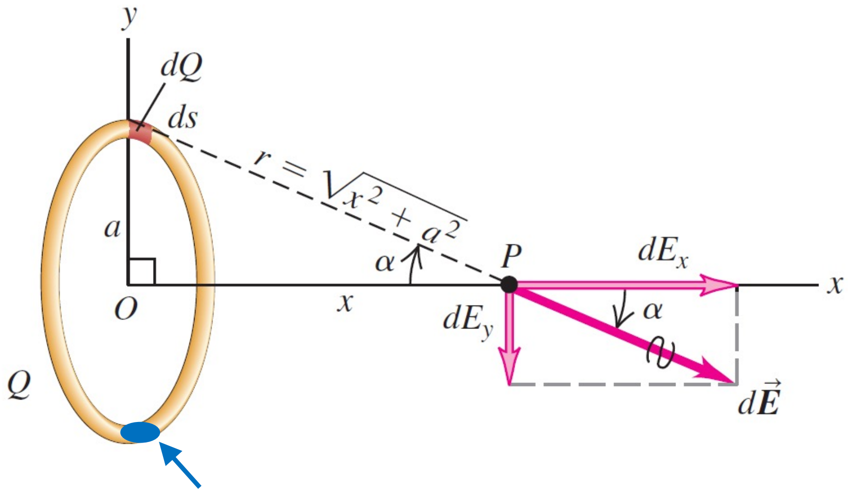
In our case, $L = 2\pi a$ and $\lambda = \frac{Q}{L} = \frac{Q}{2\pi a}$

Then the charge carried by arc length ds is

$$dQ = \lambda ds = \frac{Qds}{2\pi a}$$

Step 3 Calculate dE

$$\boxed{dQ = \lambda ds = \frac{Qds}{2\pi a}}$$



The inversion-symmetric partner of ds

Point Charge Formula

$$\begin{cases} d\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{r^2}\hat{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{x^2+a^2}\hat{r} \\[2ex] dE_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{x^2+a^2}\cos\alpha = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{x^2+a^2}\dfrac{x}{\sqrt{x^2+a^2}} \\[2ex] dE_y = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{x^2+a^2}\sin\alpha \end{cases}$$

The inversion partner contributes a same dE_x, but an opposite dE_y.

Only E_x is non-zero, as enforced by symmetry!

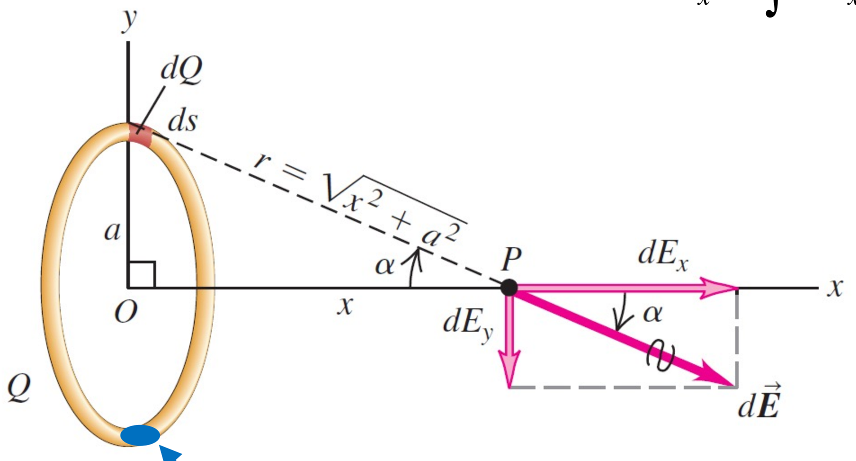
Step 4 Integration

$$dQ = \lambda ds = \frac{Qds}{2\pi a}$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}$$

$$E_x = \int dE_x = \int \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x}{\left(x^2+a^2\right)^{3/2}}\int dQ$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{x}{\left(x^2+a^2\right)^{3/2}}Q$$



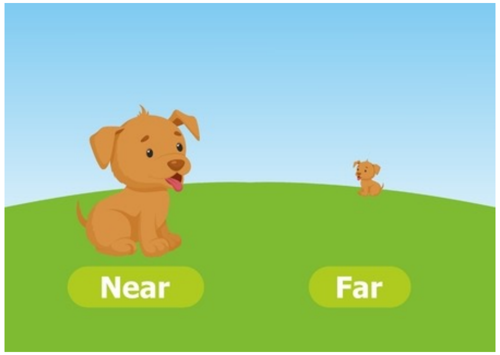
The inversion-symmetric partner of ds

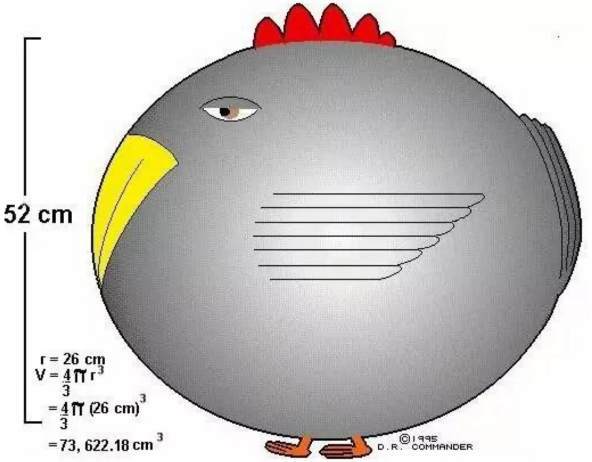
$$\int dQ = \int_0^{2\pi a} \frac{Q}{2\pi a} ds = Q$$

When $x \gg a$

Q: What does the field from the ring of charge look like if we are very far away?

A: Well, you cannot tell a ring from a point, if we are really far away…





Spherical chickens in a vacuum!

Conjecture: The field from a charged ring far away will resemble that of a point charge.

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{x}{\left(x^2+a^2\right)^{3/2}}Q = \frac{1}{4\pi\varepsilon_0}\frac{x}{x^3\left(1+a^2/x^2\right)^{3/2}}Q \xrightarrow[\text{then } a^2/x^2 \to 0]{\text{If } x^2 >> a^2} E_x = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{x^3} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$$

Electric Field of a Charged Ring

Electric Field of a Point Charge!

Other Examples

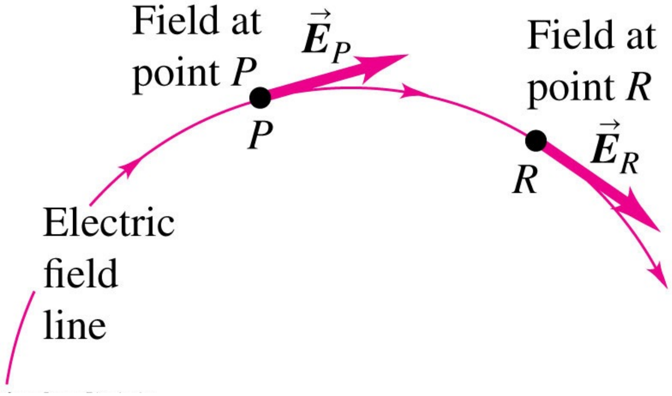
- Field of a Charged Line Segment (Example 21.10, Page 697)
- Field of a Charged Disk (Example 21.11, Page 698)
- Field of two Oppositely Charged Infinite Sheets (Example 21.12, Page 699)

These are all VERY CLASSICAL examples.

Please reproduce the calculations after the lecture.

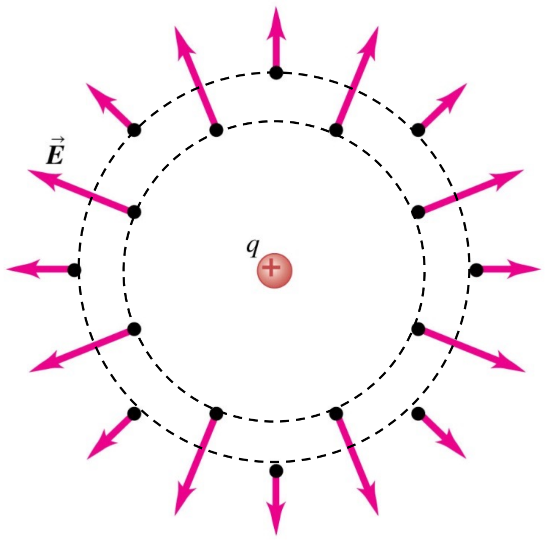
Visualization of Electric Field

An electric field line is an imaginary line or curve drawn through a region of space so that its **tangent** at any point is **in the direction of the electric field vector** at that point.



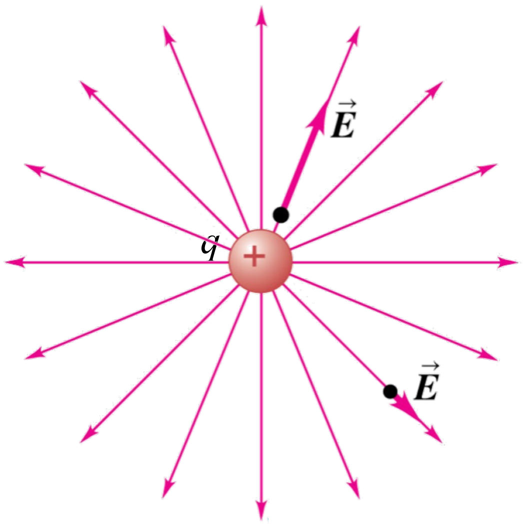
Field v.s. Field Lines

Plot of Electric Field



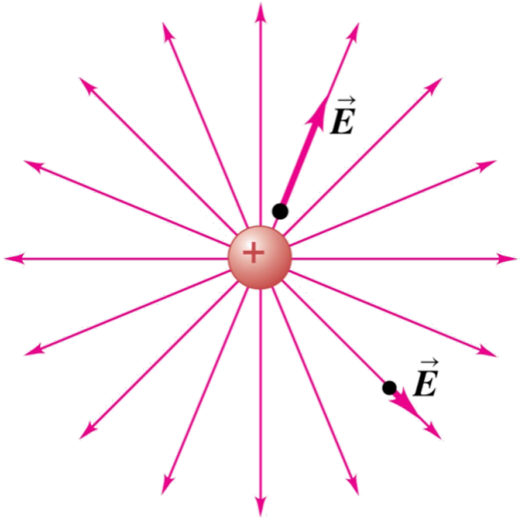
Field Magnitude = Length of arrows
Field Direction = Arrow directions

Plot of Field Line

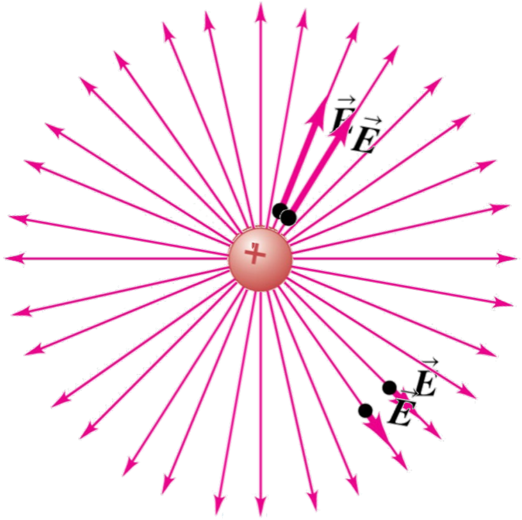


Field Magnitude = Line density/spacing
Field Direction = Tangent along the arrow

Line Density = Field Magnitude



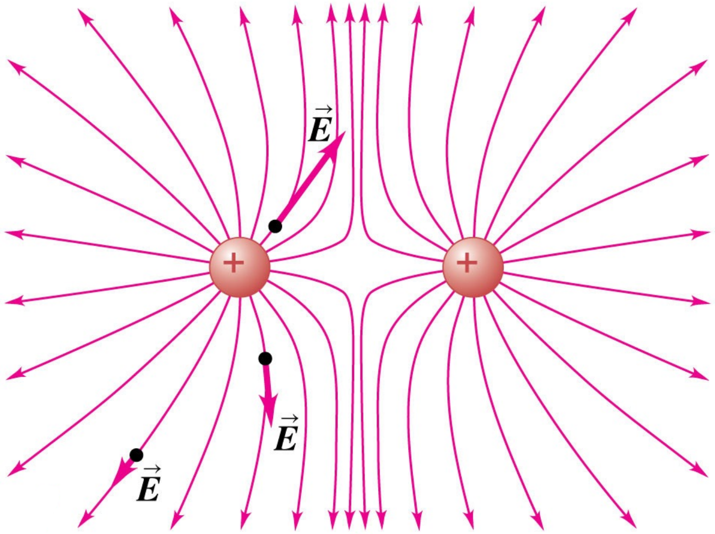
$+\boldsymbol{Q}$

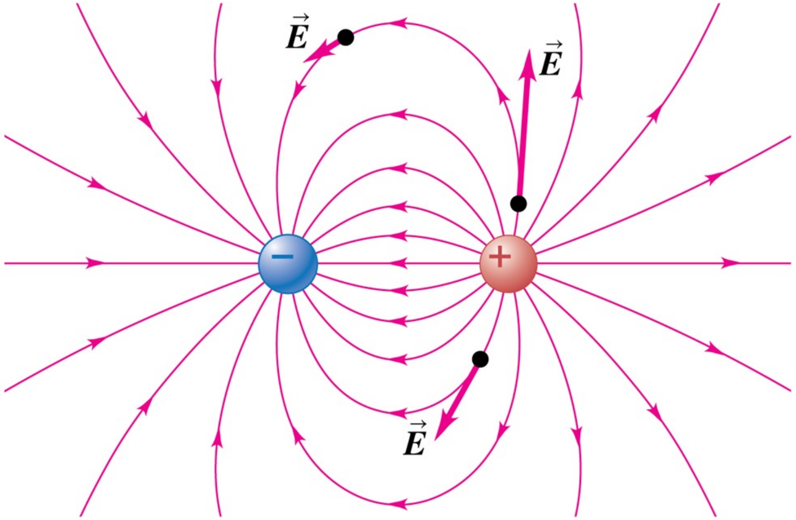


$+2\boldsymbol{Q}$

Field Magnitude = Line density/spacing
Field Direction = Tangent along the arrow

Field Lines for Multiple Charges

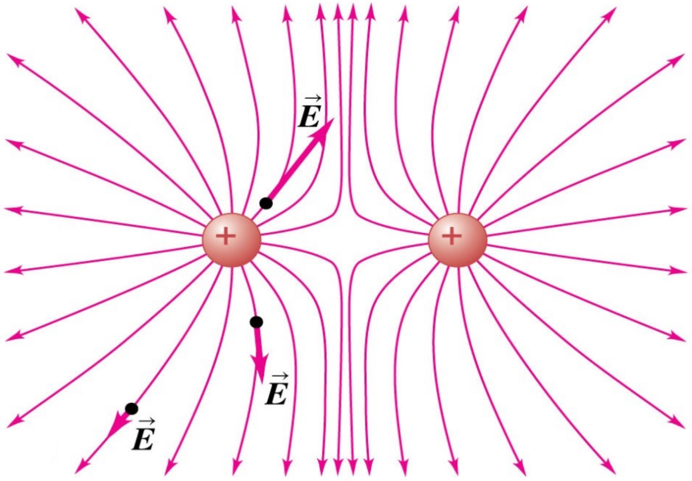


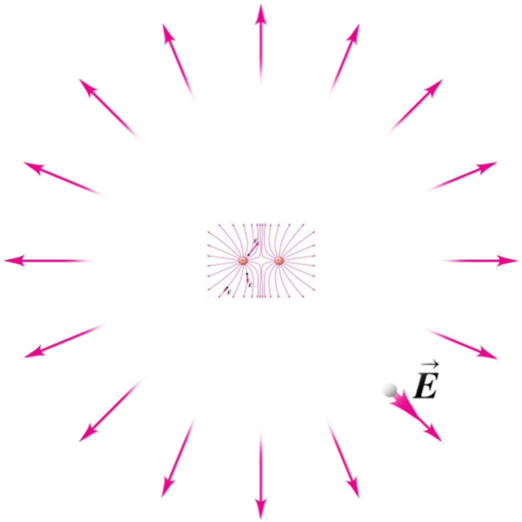


- **Field Direction = Tangent along the arrow**
- Always begin on **positive** charges.
- Always end on **negative** charges or infinity.
- **No two field lines can cross** since the field magnitude and direction must be unique.

Field Lines for Multiple Charges

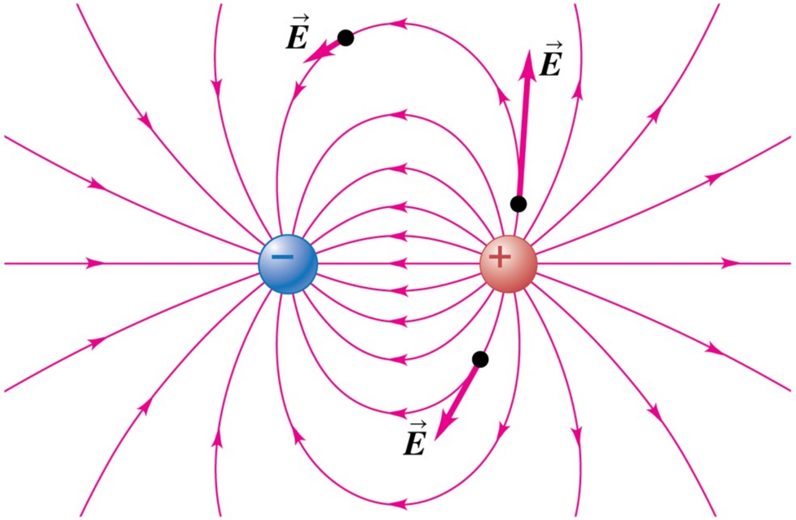
How do the electric field lines look when far far far away from the charges?

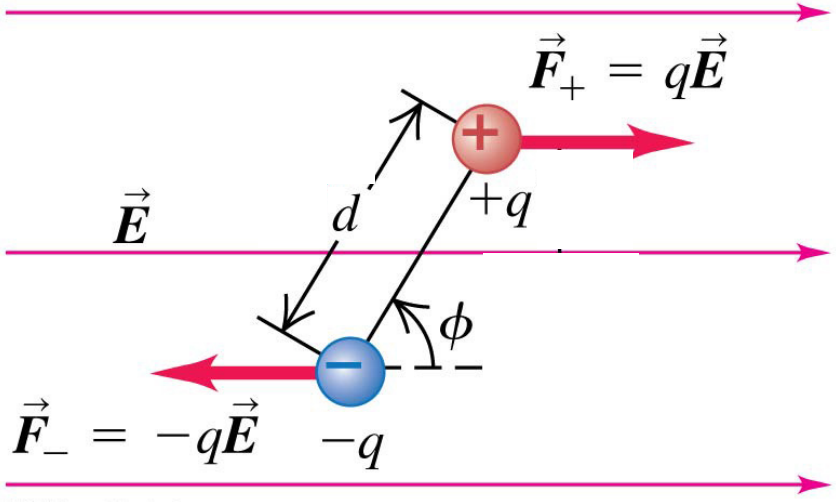




Electric Dipole

An ***electric dipole*** is a pair of particles separated by a small distance *d*, whose charges (*q* and –*q*) have *equal* magnitudes, but *opposite* signs.





The net force on an electric dipole in a **uniform** external electric field is zero.

Torque of a Dipole

The dipole tends to rotate clockwise ➡ A torque τ!

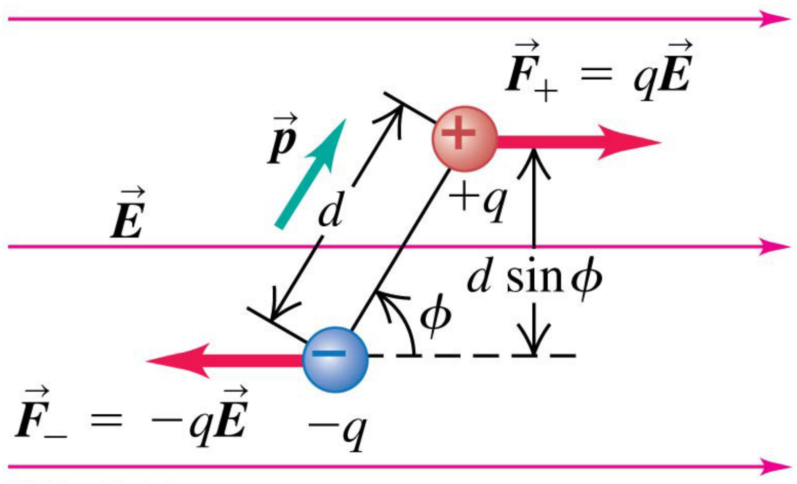
Magnitude of torque on an electric dipole

$$\tau = pE\sin\phi$$

Magnitude of electric field $\vec{E}$

Angle between $\vec{p}$ and $\vec{E}$

Magnitude of electric dipole moment $\vec{p}$

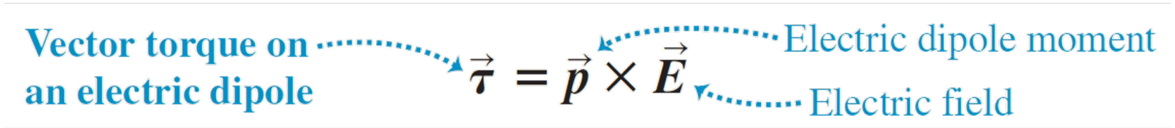


Dipole Moment $\vec{p}$

- **Magnitude:** $p = qd$
- **Direction:** From **negative** charge to **positive** charge

Torque of a Dipole

In the form of vector product

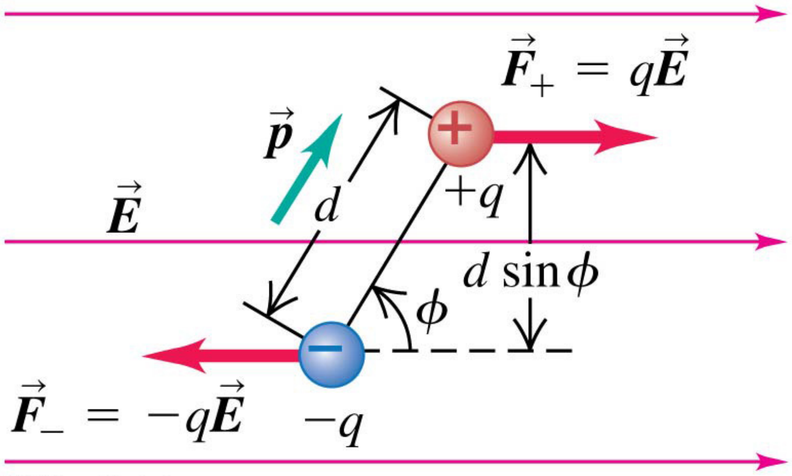


Vector torque on an electric dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Electric dipole moment

Electric field



Right-hand rule!

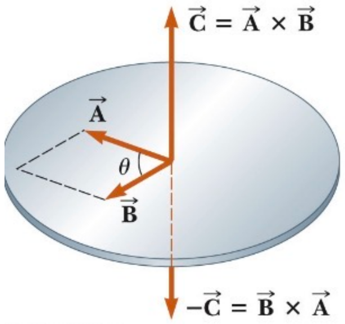
Dipole Moment $\vec{p}$

- ○ **Magnitude:** $p = qd$
- ○ **Direction:** From **negative** charge to **positive** charge

Vector Algebra

- Addition & Subtraction
- Dot Product
- Cross Product

$$|\vec{C}| = |\vec{A} \times \vec{B}| = AB\sin\theta$$



Right-hand rule

$$\begin{aligned} \mathbf{i} \times \mathbf{j} &= \mathbf{k} & \mathbf{j} \times \mathbf{i} &= -\mathbf{k} \\ \mathbf{j} \times \mathbf{k} &= \mathbf{i} & \mathbf{k} \times \mathbf{j} &= -\mathbf{i} \\ \mathbf{k} \times \mathbf{i} &= \mathbf{j} & \mathbf{i} \times \mathbf{k} &= -\mathbf{j} \end{aligned}$$

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

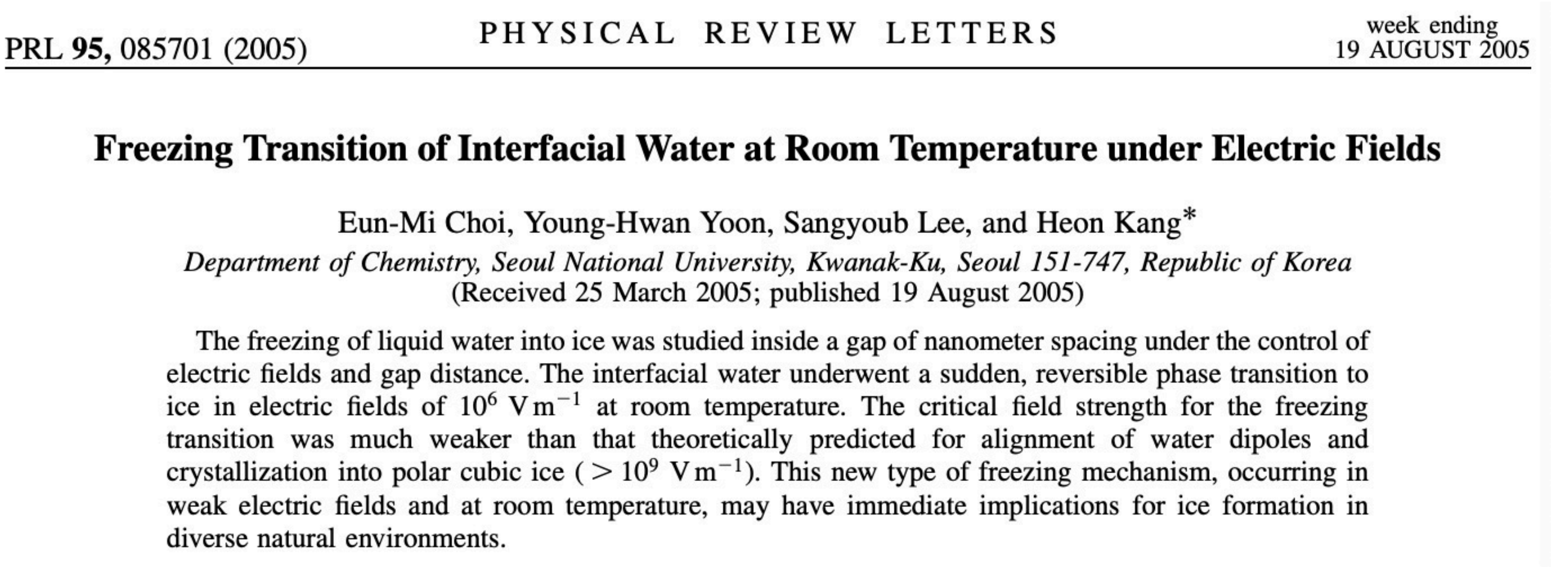
$$\begin{aligned} \mathbf{A} \times \mathbf{B} =& (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \times (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) \\ =& a_1b_1(\mathbf{i} \times \mathbf{i}) + a_1b_2(\mathbf{i} \times \mathbf{j}) + a_1b_3(\mathbf{i} \times \mathbf{k}) + \\ & a_2b_1(\mathbf{j} \times \mathbf{i}) + a_2b_2(\mathbf{j} \times \mathbf{j}) + a_2b_3(\mathbf{j} \times \mathbf{k}) + \\ & a_3b_1(\mathbf{k} \times \mathbf{i}) + a_3b_2(\mathbf{k} \times \mathbf{j}) + a_3b_3(\mathbf{k} \times \mathbf{k}) \\ =& (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k} \end{aligned}$$

or

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Remember: Always do calculation at coordinate basis.

Electric Dipole in real world



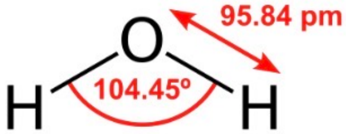
PRL **95,** 085701 (2005) PHYSICAL REVIEW LETTERS week ending 19 AUGUST 2005

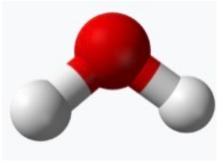
Freezing Transition of Interfacial Water at Room Temperature under Electric Fields

Eun-Mi Choi, Young-Hwan Yoon, Sangyoub Lee, and Heon Kang*
Department of Chemistry, Seoul National University, Kwanak-Ku, Seoul 151-747, Republic of Korea

(Received 25 March 2005; published 19 August 2005)

The freezing of liquid water into ice was studied inside a gap of nanometer spacing under the control of electric fields and gap distance. The interfacial water underwent a sudden, reversible phase transition to ice in electric fields of 10^6 V m^{-1} at room temperature. The critical field strength for the freezing transition was much weaker than that theoretically predicted for alignment of water dipoles and crystallization into polar cubic ice ($> 10^9$ V m^{-1}). This new type of freezing mechanism, occurring in weak electric fields and at room temperature, may have immediate implications for ice formation in diverse natural environments.





Spoiler for Next Lecture

Charles-Augustin de Coulomb
(1736 - 1806)

Johann Carl Friedrich Gauss
(1777 - 1855)