ELECTRICITY AND MAGNETISM
(PHYS 231)

Lecture 2: Electric Charge & Coulomb's Law

Aug 21, 2024

Homework Due 11 pm Aug. 28
Lab starts next week

Office Hour & Tutorial Center

1) Office Hour

Location: 217A NIELSEN

Time: 09:30 – 10:30 am every Monday

2) Tutorial Center

Location: 512 Nielsen Building

Time: (Almost ANYTIME between 11:15 am to 3:25 pm, Monday to Friday)

3) On-line TA: Louis Primeau

Canvas, clicker, homework system related questions.

Review on Vectors

- Scalar: a quantity has only magnitude.
- Vector: a quantity has both magnitude & **direction**.

Cartesian Coordinate

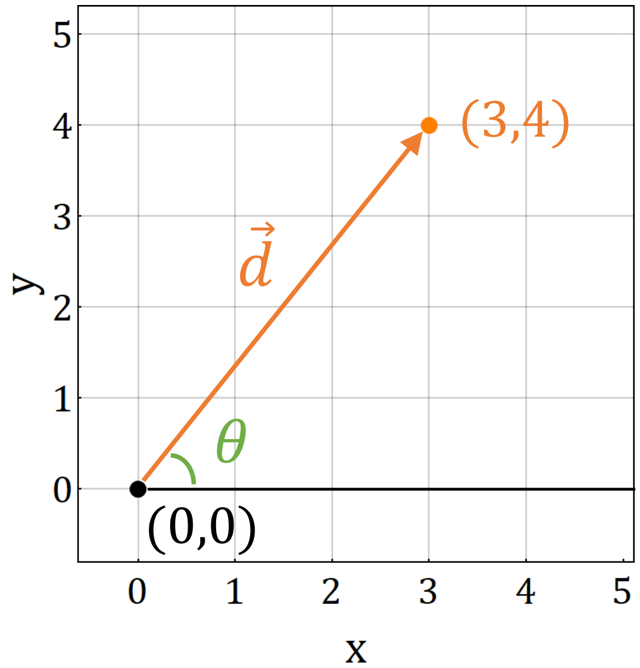
$$\vec{d} = d_x\, \boldsymbol{i} + d_y\, \boldsymbol{j} = 3\, \boldsymbol{i} + 4\, \boldsymbol{j}$$

$$\vec{d} = \left(d_x, d_y\right) = (3, 4)$$

Polar Coordinate

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{3^2 + 4^2} = 5 \qquad \text{(magnitude)}$$

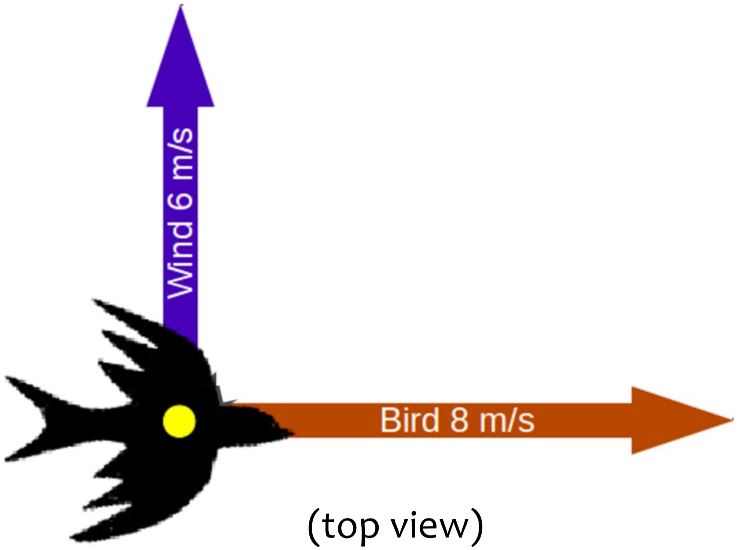
$$\tan\theta = y/x, \qquad \theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 0.295\,\pi \qquad \text{(direction)}$$



Note: The polar angle θ is always defined **counter-clockwise** with respect to the x axis

Vector Algebra

- Addition & Subtraction



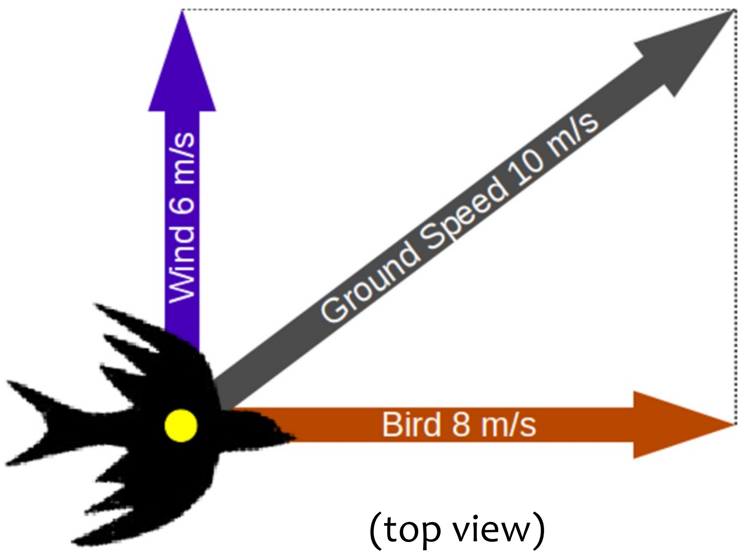
(top view)

The bird plans to fly along the red arrow, but…

- Dot Product
- Cross Product

Vector Algebra

- Addition & Subtraction



(top view)

The bird plans to fly along the red arrow, but…

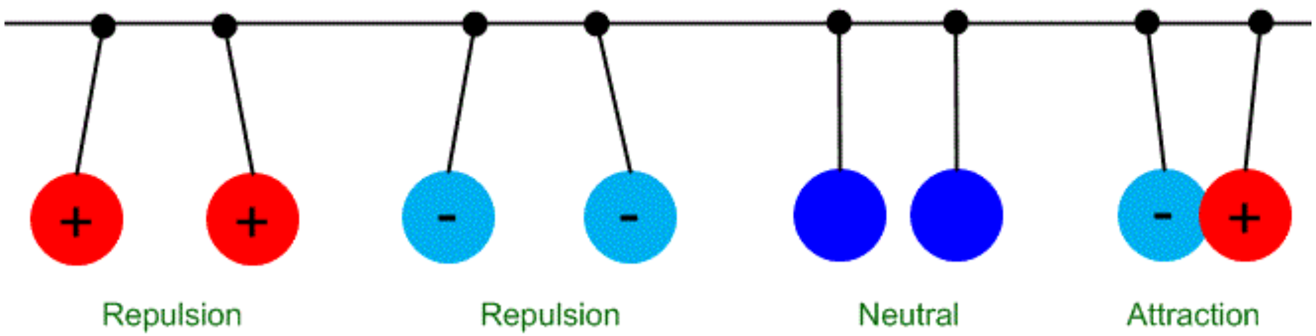
- Dot Product
- Cross Product

Electric Charge

- **Electric Charge:** 1) a physical property of matter
  2) with which matter can interact electromagnetically

- Electric charge can be either positive or negative.

- Two positive charges or two negative charges **repel** each other. A positive charge and a negative charge **attract** each other.



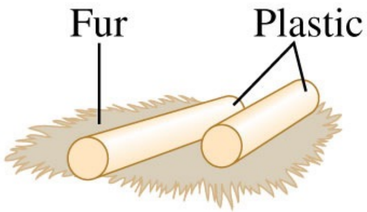
The Laws of Attraction and Repulsion

More on Charge

- Charge can add like signed numbers. (e.g. 1+(-1)=0)
- Charge cannot be created or destroyed.
- The net charge in an isolated system is **conserved**.
  (Law of Charge Conservation)
- Conventional symbol for charge: $\pm Q$ or $\pm q$.
- The SI Unit of charge is "Coulomb", denoted as C

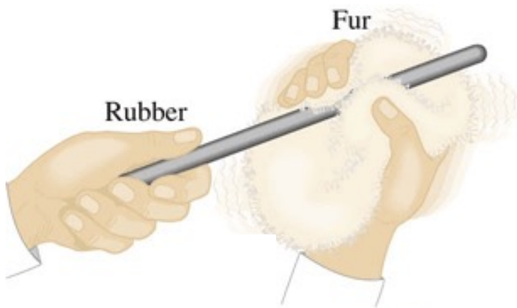
Charles-Augustin de Coulomb
(1736 - 1806)

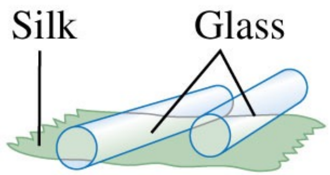
Example



Rub the plastic rod

Initially charge neutral

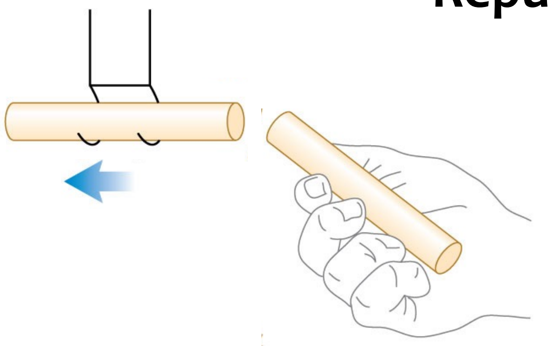


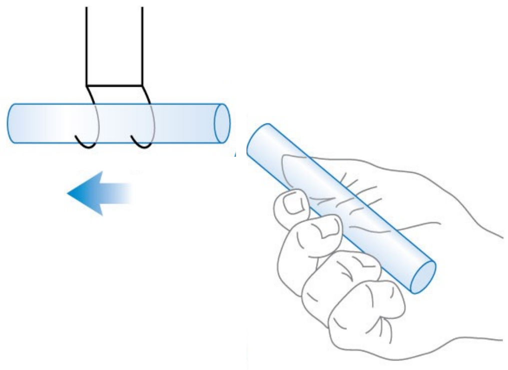


Rub the glass rod

Do the same experiment but with two plastic/glass rods.

Repulsive Electric Force





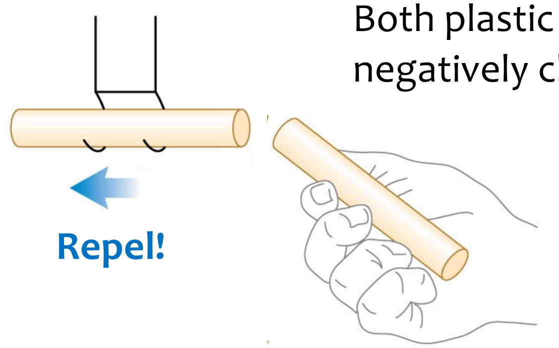
Example

Different materials have different capabilities of gaining charges.

Fur < Plastic & Silk > Glass

Charge transfer happens when fur & plastic (or silk and glass) have contact.

After rubbing:
- Fur (positively charged)
- Plastic (negatively charged)
- Silk (negatively charged)
- Glass (positively charged)

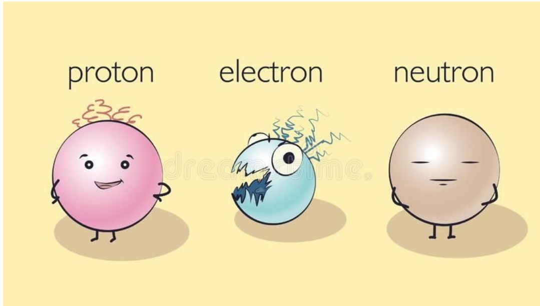


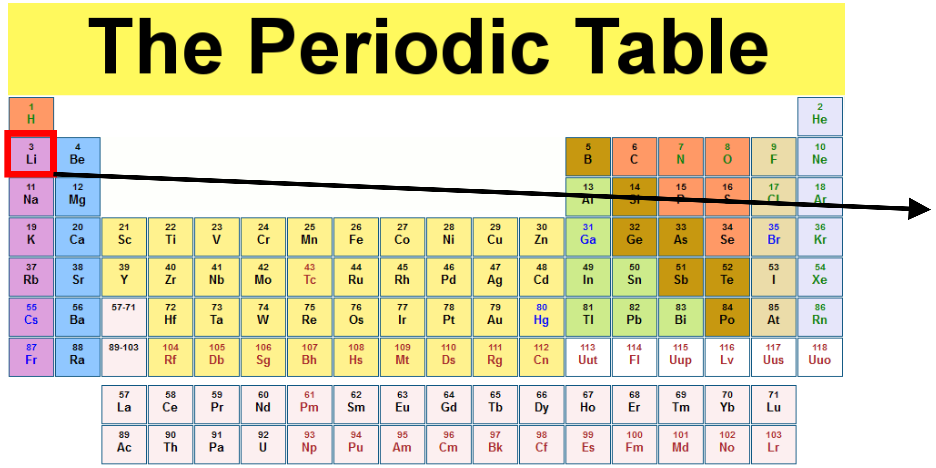
Both plastic rods are negatively charged.

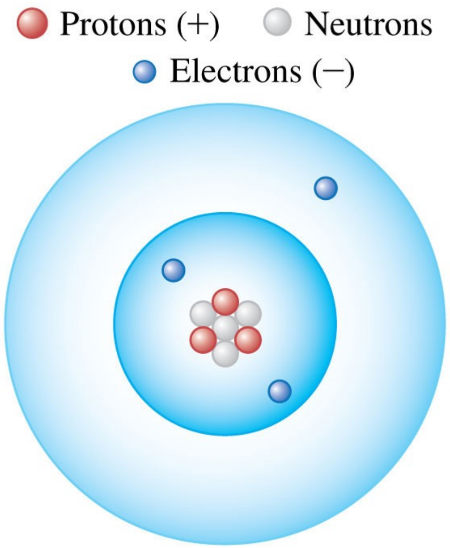
Every material has a "bank" of "+" & "–" charges!

Microscopic Origin of Charge

- Matter is made up of **Atoms** (nucleus & electrons)
- Atoms are electrically neutral objects
- The Nucleus is made up of protons and neutrons







Neutral lithium atom (Li):

3 protons (3+)

4 neutrons

3 electrons (3−)

Electrons equal protons:
Zero net charge

Microscopic Origin of Charge

Electron

The elementary subatomic building block for negative charges.

Mass of an electron: $m_e = 9.109 * 10^{-31}$ kg

Charge of an electron: $q_e \equiv -1.602 * 10^{-19}\ C$

Proton

The elementary subatomic building block for positive charges.

Mass of a proton: $m_p = 1.673 * 10^{-27}$ kg

Charge of a proton: $q_p \equiv -q_e = +1.602 * 10^{-19}\ C$

Atom consists of equal #s of electrons & protons

⇩

Charge Neutral

$m_p \gg m_e$ ⇨ charge transfer is mostly electron transfer.

A neutral atom
- Loses electrons → **Positive** Ion e.g. Li^{+} (lose 1 e) or Li^{2+} (lose 2e)
- Gains electrons → **Negative** Ion e.g. Li^{-} (gain 1 e) or Li^{2-} (gain 2e)

Conductors & Insulators

Conductors permit the easy movement of charge through them, while insulators do not.

Conductors v.s. **Insulators**

(e.g. metals) (e.g. rubber, glass)

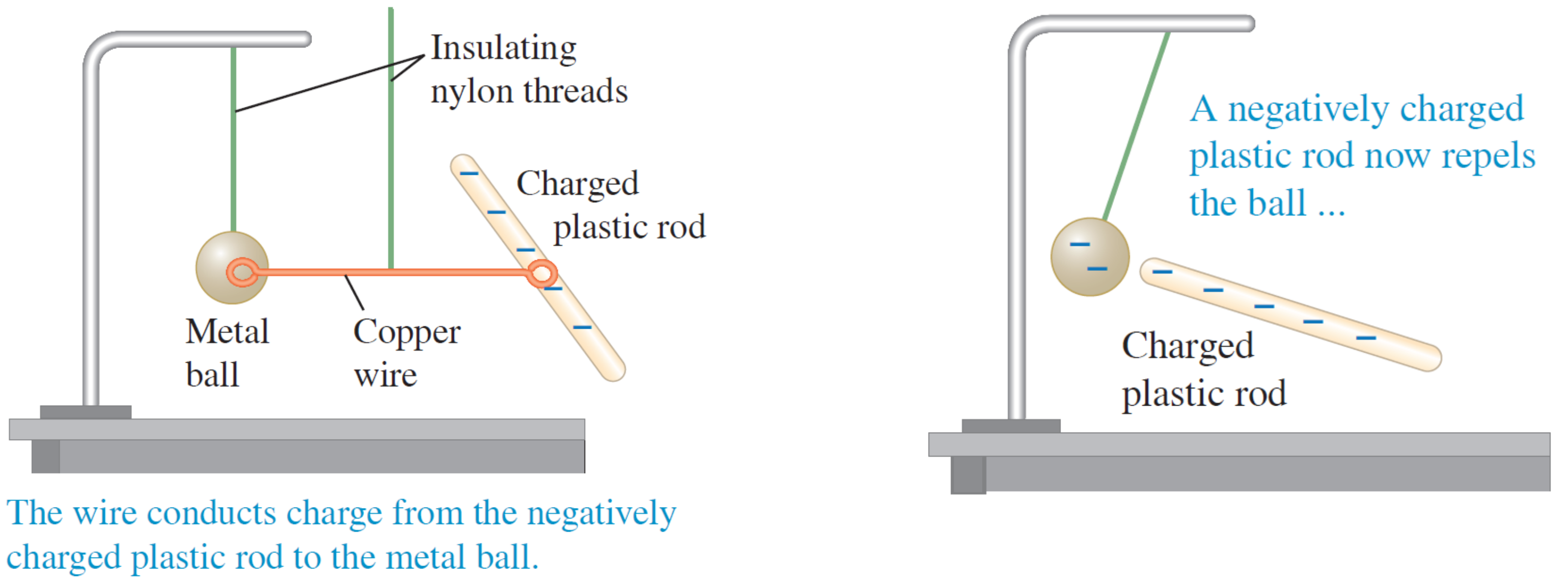
Metals (e.g. copper) are good conductors.

The ONLY perfect conductors are the **superconductors** at extremely low temperature.

There is NO perfect insulator.

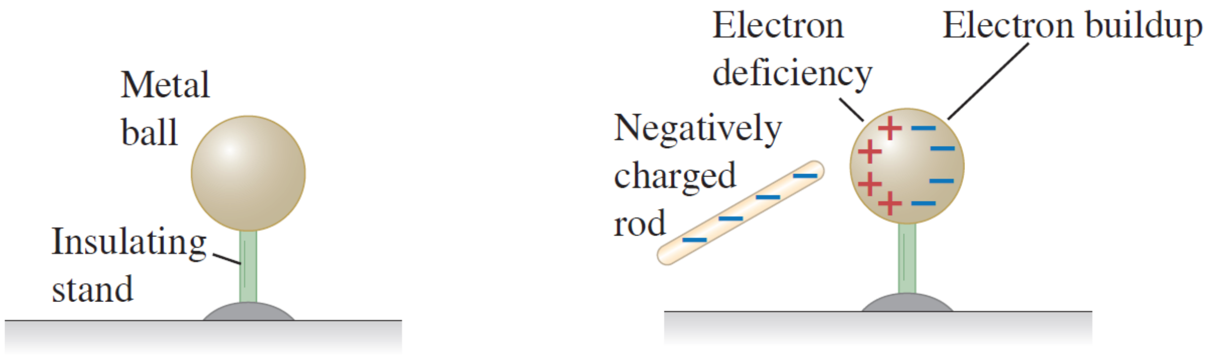
My own research focuses on **superconductor** and **topological insulators**, a class of exotic materials whose bulk is insulating, while the outer surface is metallic…

Conduction



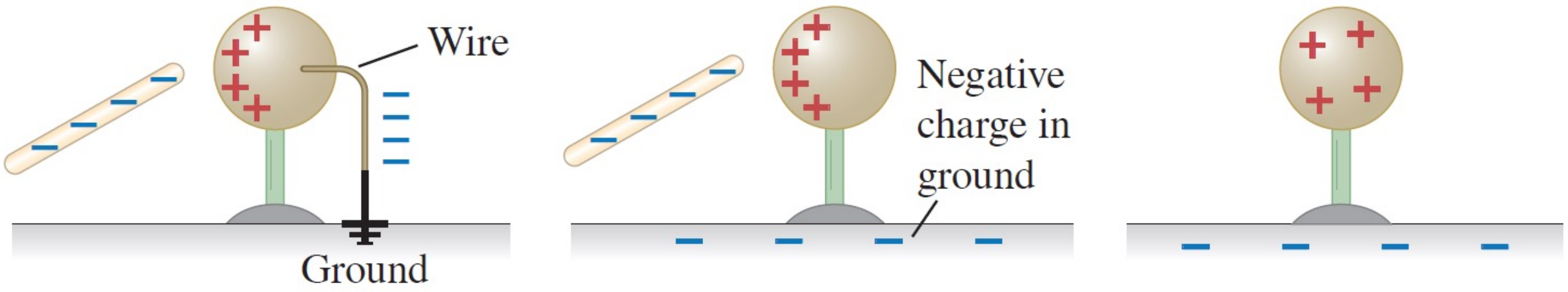
Charging a metal ball by **conduction**.

Induced Charge



1) Starting with a charge neutral metal ball.
2) Negative charge on the rod repels electrons of the metal ball.
3) Metal ball forms zones of positive & negative **induced charge**.
4) The metal ball **attracts** the charged rod.

Induction



Charging a neutral metallic ball via **induction**.

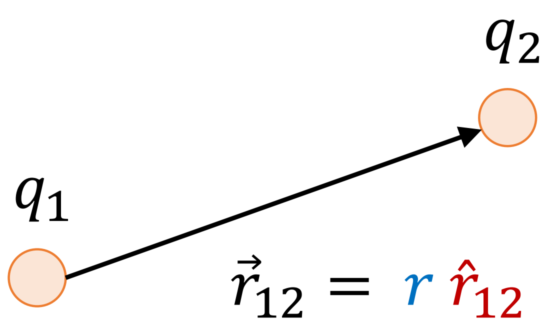
Q: Why a charged object can attract an insulating object?

A: See Fig. 21.8 for details.

Quantify the Attraction/Repulsion Force

Coulomb's Law

$$\vec{F}_{1\to 2} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$



- $\vec{F}_{1\to 2}$ is the electric force of q_1 on q_2.
- $k = 8.988\times 10^9\ Nm^2/C^2$ is the **Coulomb** constant.
- $k \equiv 1/(4\pi\varepsilon_0)$, where ε_0 is the **electric** constant. The formula with k or ε_0 will be used interchangeably.
- Attractive force: q_1 & q_2 have opposite signs.
- Repulsive force: q_1 & q_2 share the same sign.

Magnitude: $r = |\vec{r}_{12}|$

Direction: $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$
(unit vector)

The Ultimate Tips for Calculating Coulomb Force

Step 1

Identify r and $\hat{r}_{12}$

Step 2

Calculate the magnitude of $\vec{F}_{1\to 2}$ following

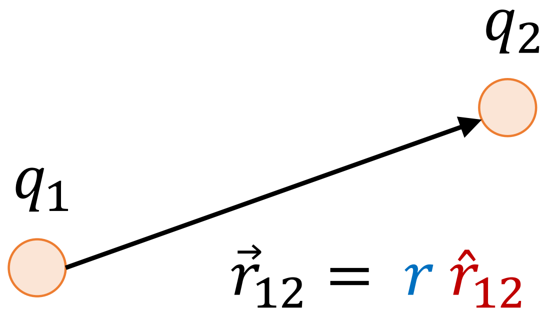
$$|F_{1\to 2}| = k\,\frac{|q_1 q_2|}{r^2}$$

Step 3

The direction of $\vec{F}_{1\to 2}$ is determined by

a) $+\hat{r}_{12}$ if $q_1 q_2 > 0$;

b) $-\hat{r}_{12}$ if $q_1 q_2 < 0$.



$$\vec{r}_{12} = r\,\hat{r}_{12}$$

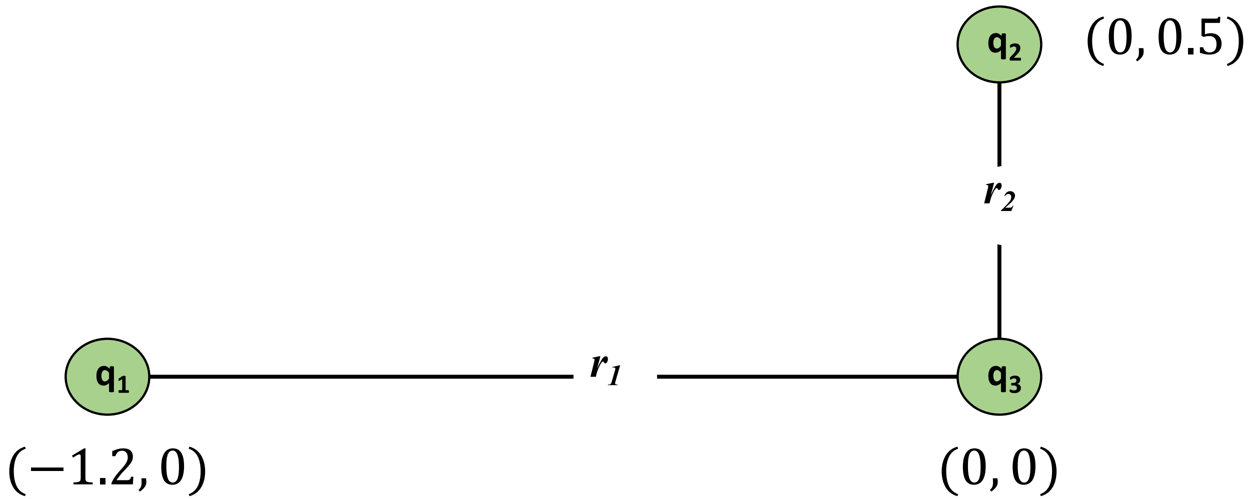
Magnitude: $r = |\vec{r}_{12}|$

Direction: $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$
(unit vector)

Example Problem

Given the layout below, find the force on charge q_3

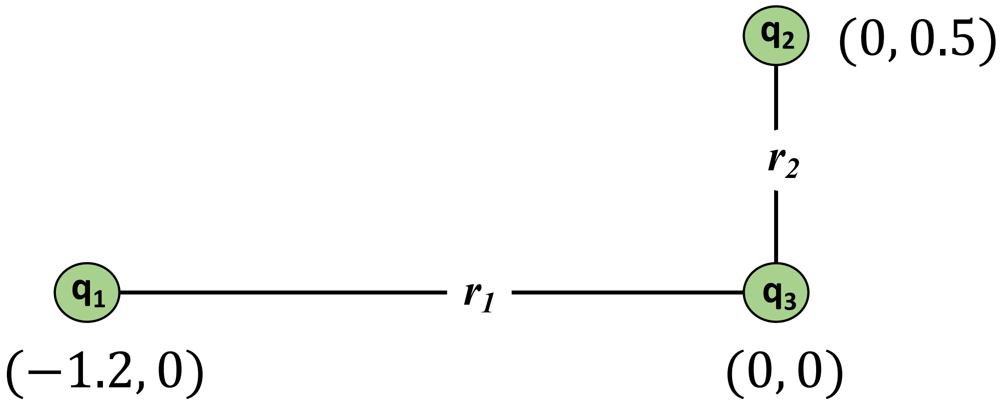
$q_1 = +1.5\times10^{-3}C$
$q_2 = -0.5\times10^{-3}C$
$q_3 = +0.2\times10^{-3}C$



Step 1: Identify $r_{13}, \hat{r}_{13}, r_{23}, \hat{r}_{23}$

$$\vec{r}_{13} = \vec{r}_3 - \vec{r}_1 = (1.2, 0) = 1.2\,\boldsymbol{i}$$
$$\vec{r}_{23} = (0, -0.5) = -0.5\,\boldsymbol{j}$$

$$r_{13} = 1.2, \qquad r_{23} = 0.5$$
$$\hat{r}_{13} = \boldsymbol{i}, \qquad \hat{r}_{23} = -\boldsymbol{j}$$

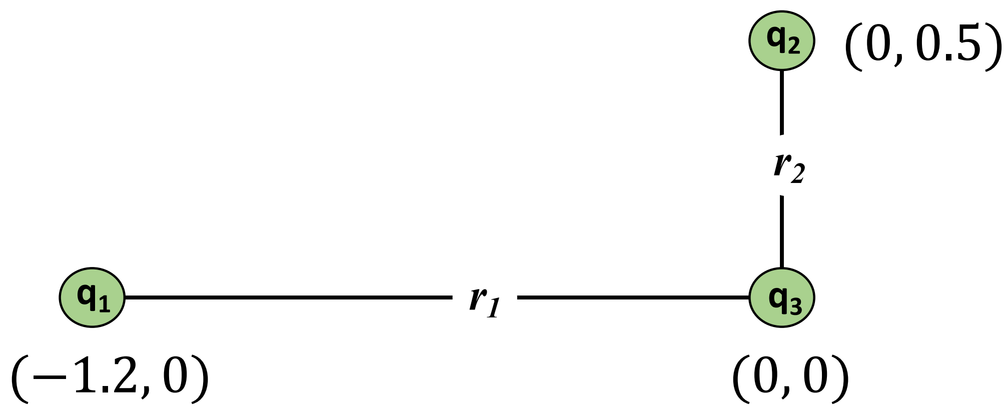


Step 2: Calculate the Force magnitude

$$r_{13} = 1.2, \quad r_{23} = 0.5$$
$$\hat{r}_{13} = \boldsymbol{i}, \quad \hat{r}_{23} = -\boldsymbol{j}$$

$$|F_{1\to3}| = k\frac{|q_1 q_3|}{r_{13}^2} = 1.88\times10^3 N$$

$$|F_{2\to3}| = k\frac{|q_2 q_3|}{r_{23}^2} = 3.60\times10^3 N$$



Step 3: Identify Force Direction

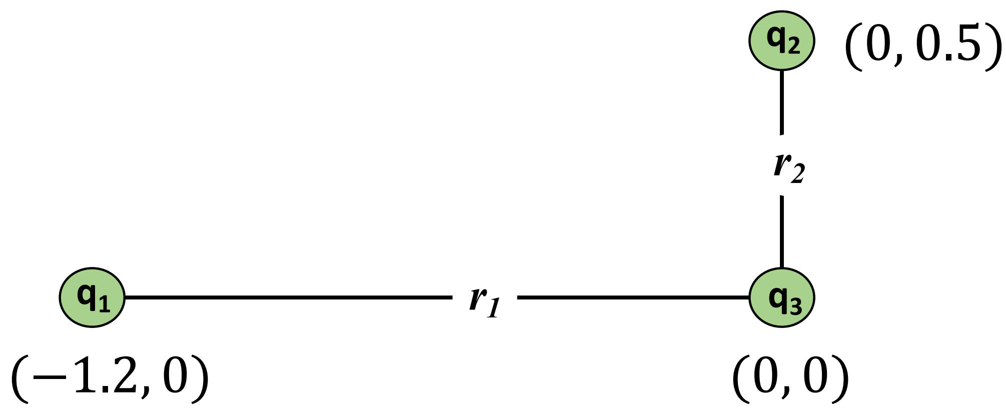
$$r_{13} = 1.2, \qquad r_{23} = 0.5$$
$$\hat{r}_{13} = \boldsymbol{i}, \qquad \hat{r}_{23} = -\boldsymbol{j}$$

$$|F_{1\to3}| = k\frac{|q_1 q_3|}{r_{13}^2} = 1.88\times10^3 N$$
$$|F_{2\to3}| = k\frac{|q_2 q_3|}{r_{23}^2} = 3.60\times10^3 N$$

$$q_1 = +1.5\times10^{-3} C$$
$$q_2 = -0.5\times10^{-3} C$$
$$q_3 = +0.2\times10^{-3} C$$

$$\vec{F}_{1\to3} = |F_{1\to3}|(+\hat{r}_{13}) = (+1.88\times10^3 N)\ \boldsymbol{i}$$
$$\vec{F}_{2\to3} = |F_{2\to3}|(-\hat{r}_{23}) = (+3.6\times10^3 N)\ \boldsymbol{j}$$

The minus sign comes from $q_2 q_3 < 0$.



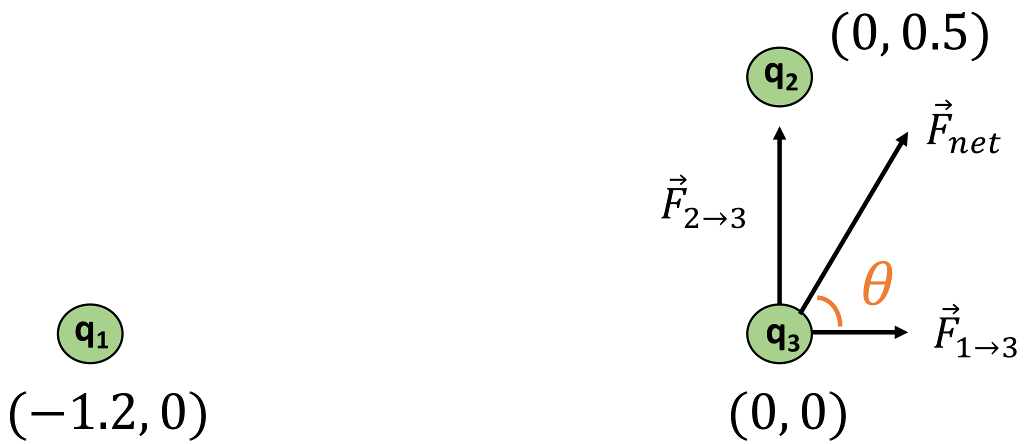
Step 4: Superposition of Forces

$$\vec{F}_{1\to3} = |F_{1\to3}|(+\hat{r}_{13}) = (+1.88\times10^{3}N)\ \boldsymbol{i}$$

$$\vec{F}_{2\to3} = |F_{2\to3}|(-\hat{r}_{23}) = (+3.6\times10^{3}N)\ \boldsymbol{j}$$

$$\vec{F}_{net} = \vec{F}_{1\to3} + \vec{F}_{2\to3}$$

$$= (1880\ N, 3600N)$$



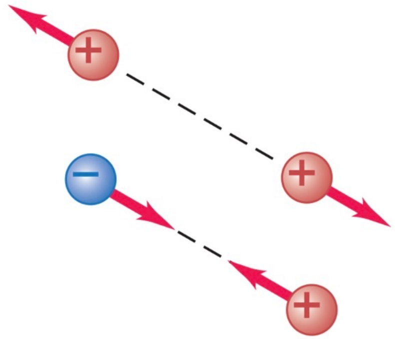
$$|F_{net}| = \sqrt{F_{13}^2 + F_{23}^2} = 4.06\times10^{3}N$$

$$\theta = \tan^{-1}\frac{3600}{1880} \approx 62.4°$$

Summary

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.



Conductors are materials in which charge moves easily; in insulators, charge does not move easily. Most metals are good conductors; most nonmetals are insulators.

Why electrons do not fall into nucleus?

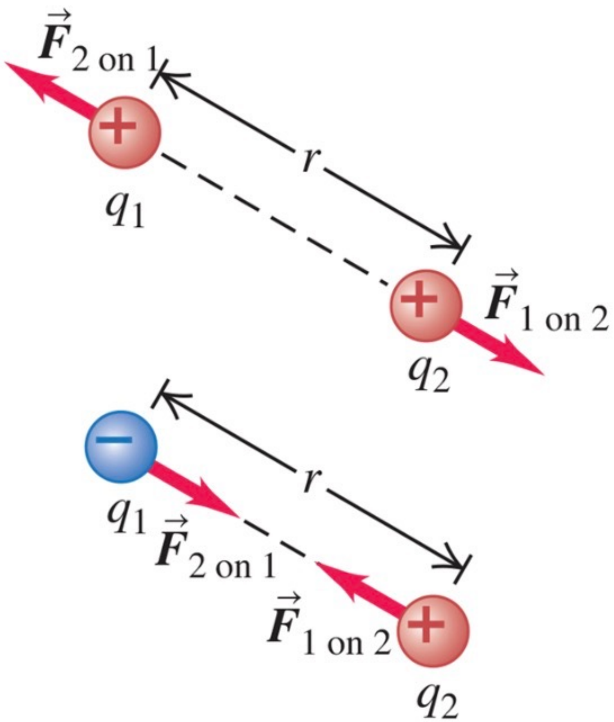
Summary

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

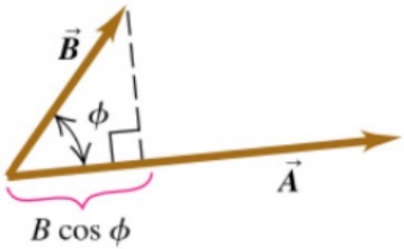
Why charges with different sign attract? (sign of Coulomb's Law)

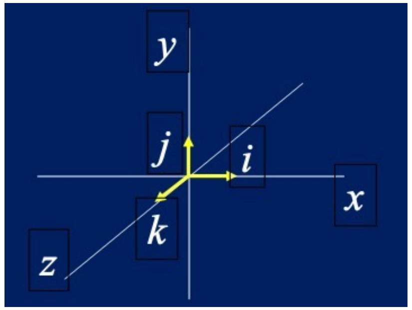
Coulomb's law



Vector Algebra

- Addition & Subtraction
- Dot Product





$$\vec{A} = \mathrm{A_x}\hat{\imath} + \mathrm{A_y}\hat{\jmath} + \mathrm{A_z}\hat{k}$$
$$\vec{B} = \mathrm{B_x}\hat{\imath} + \mathrm{B_y}\hat{\jmath} + \mathrm{B_z}\hat{k}$$
$$\vec{A} \cdot \vec{B} = \mathrm{A_x B_x} + \mathrm{A_y B_y} + \mathrm{A_z B_z}$$

$$\vec{A} \cdot \vec{B} = |A||B| \cos\varphi = AB\cos\varphi$$

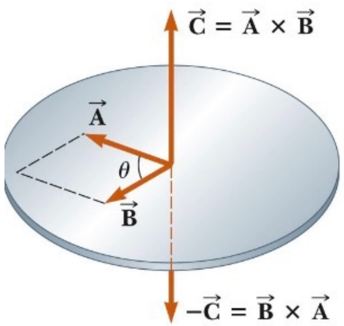
The dot product of two vectors says something about how parallel they are.

- Cross Product

Vector Algebra

- Addition & Subtraction
- Dot Product
- Cross Product

$$|\vec{C}| = |\vec{A} \times \vec{B}| = AB \sin\theta$$



Right-hand rule

$$\begin{aligned} \mathbf{i} \times \mathbf{j} &= \mathbf{k} & \mathbf{j} \times \mathbf{i} &= -\mathbf{k} \\ \mathbf{j} \times \mathbf{k} &= \mathbf{i} & \mathbf{k} \times \mathbf{j} &= -\mathbf{i} \\ \mathbf{k} \times \mathbf{i} &= \mathbf{j} & \mathbf{i} \times \mathbf{k} &= -\mathbf{j} \end{aligned}$$

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

$$\begin{aligned} \mathbf{A} \times \mathbf{B} =& (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \times (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) \\ =& a_1b_1(\mathbf{i} \times \mathbf{i}) + a_1b_2(\mathbf{i} \times \mathbf{j}) + a_1b_3(\mathbf{i} \times \mathbf{k}) + \\ & a_2b_1(\mathbf{j} \times \mathbf{i}) + a_2b_2(\mathbf{j} \times \mathbf{j}) + a_2b_3(\mathbf{j} \times \mathbf{k}) + \\ & a_3b_1(\mathbf{k} \times \mathbf{i}) + a_3b_2(\mathbf{k} \times \mathbf{j}) + a_3b_3(\mathbf{k} \times \mathbf{k}) \\ =& (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k} \end{aligned}$$

or

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Remember: Always do calculation at coordinate basis.