Department of Mathematics – University of Tennessee

Math 251 Matrix Algebra I

Test 3

Name: ______________________________

Time allowed: 50 minutes

Instructions:

- Calculators are not allowed.
- All electronic devices must be put away.
- Answers with insufficient or incorrect working will not receive full credit.
- Simplify answers whenever possible.

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	50	

1. (10 pts) Decide whether each statement is TRUE or FALSE. *No justification is required.*

   - The set of all polynomials with the usual addition and scalar multiplication is a vector space.
   - The intersection of two subspaces of a vector space V is also a subspace of V.
   - If S is a set of vectors in a vector space V, then $\operatorname{span}(S)$ is a subspace of V.
   - The span of a nonzero vector in $\mathbb{R}^3$ is a line.
   - The polynomials $x-1$, $(x-1)^2$, and $(x-1)^3$ span P_3.
   - If W is a set of vectors containing the zero vector, then W is linearly independent.
   - Every set of n linearly independent vectors in $\mathbb{R}^n$ is a basis for $\mathbb{R}^n$.
   - If B is the standard basis for $\mathbb{R}^n$ and $\vec{v}$ is in $\mathbb{R}^n$, then $[\vec{v}]_B = \vec{v}$.
   - If W is a subspace of a finite-dimensional vector space V, then $\dim W \leq \dim V$.
   - If A is a 3×3 matrix, then $A = P_{B_1 \to B_2}$ for some bases B_1 and B_2 of $\mathbb{R}^3$.

2. (6 pts) Let V be the set of all pairs $\vec{v} = (a, b)$ of real numbers with the following operations:

$$(a, b) + (c, d) = (a + c, 0),$$
$$k(a, b) = (ka, kb).$$

(a) Does V satisfy axiom 2 for vector spaces? Prove or give a counterexample.

(b) Does V satisfy axiom 8 for vector spaces? Prove or give a counterexample.

3. (4 pts) Determine whether the set of polynomials of the form $p(x) = a + bx^2$ is a subspace of P_2.

4. (10 pts) Consider the matrices $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

   (a) Determine whether A, B, and C span M_{22}.

   (b) Determine whether A, B, and C are linearly independent in M_{22}.

   (c) Is the set $\{A, B, C\}$ a basis for M_{22}? Explain briefly.

5. (10 pts) Let $B_1 = \{(1,1),(1,-1)\}$ and $B_2 = \{(0,1),(1,2)\}$ be two bases for $\mathbb{R}^2$.

   (a) Find the coordinate vector $[\vec{v}]_{B_1}$ with respect to B_1 for $\vec{v} = (2,4)$.

   (b) Find the transition matrix $P_{B_1 \to B_2}$.

   (c) Use your answer from part (a) to find $[\vec{v}]_{B_2}$.

   (d) Does your answer to part (c) match what you expect? Explain briefly.

6. (6 pts) Consider the linear system below.

$$\begin{aligned} x_1 - 2x_2 + 2x_3 &= 0 \\ x_1 - x_2 + 2x_3 - x_4 &= 0 \\ x_2 - x_4 &= 0 \end{aligned}$$

The general solution for this system is $x_1 = -2s + 2t$, $x_2 = t$, $x_3 = s$, $x_4 = t$.

(a) Find a basis for the solution space of the linear system.

(b) What is the dimension of this solution space? Explain briefly.

7. (4 pts) State the dimension of each vector space. *No justification is required.*

(a) $\mathbb{R}^5$

(b) P_3

(c) The set of all 3×3 lower triangular matrices.

(d) The subspace of $\mathbb{R}^3$ spanned by the vectors $\vec{u} = (1, -1, 0)$ and $\vec{v} = (-1, 1, 0)$.