Department of Mathematics – University of Tennessee

Math 251 Matrix Algebra I

Test 3 Practice

Name: ______________________________

Time allowed: 50 minutes

Instructions:

- Calculators are not allowed.
- All electronic devices must be put away.
- Answers with insufficient or incorrect working will not receive full credit.
- Simplify answers whenever possible.

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	50	

1. (10 pts) Decide whether each statement is TRUE or FALSE. *No justification is required.*

   - $\mathbb{R}^n$ with the usual addition and scalar multiplication is a vector space.
   - A vector space must contain at least two vectors.
   - The set of all invertible $n \times n$ matrices is a subspace of M_{nn}.
   - The union of two subspaces of a vector space V is also a subspace of V.
   - The span of a finite set of vectors in a vector space V is closed under addition.
   - Every linearly independent set contains the zero vector.
   - If $V = \text{span}(S)$, then S is a basis for V.
   - Every linearly independent subset of a vector space V is a basis for V.
   - Every set of four vectors that span $\mathbb{R}^4$ is a basis for $\mathbb{R}^4$.
   - Transition matrices are invertible.

2. (6 pts) Let V be the set of all pairs $\vec{v} = (a, b)$ of real numbers with the following operations:

$$(a, b) + (c, d) = (a + c, b - d),$$
$$k(a, b) = (ka, 0).$$

(a) Does V satisfy axiom 2 for vector spaces? Prove or give a counterexample.

(b) Does V satisfy axiom 8 for vector spaces? Prove or give a counterexample.

3. (4 pts) Determine whether the set of 2×2 matrices $\begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$ such that $x + z = 0$ is a subspace of M_{22}.

4. (10 pts) Let $S = \{1, 1 + x, 1 - x + x^2\}$.

   Recall that P_2 is the set of all polynomials with degree at most 2.

   (a) Determine whether S spans P_2.

   (b) Determine whether S is linearly independent in P_2.

   (c) Is S a basis for P_2? Explain briefly.

5. (10 pts) Let $B_1 = \{(1,0),(1,1)\}$ and $B_2 = \{(0,1),(1,-1)\}$ be two bases for $\mathbb{R}^2$.

   (a) Find the coordinate vector $[\vec{v}]_{B_1}$ with respect to B_1 for $\vec{v} = (3,1)$.

   (b) Find the transition matrix $P_{B_1 \to B_2}$.

   (c) Use your answer from part (a) to find $[\vec{v}]_{B_2}$.

6. (10 pts) Describe the solution space for each linear system as a subspace of $\mathbb{R}^3$, and state the dimension of each solution space.

(a) $$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(c) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$