Department of Mathematics – University of Tennessee

Math 251 Matrix Algebra I

Test 2

Name: ____________________

Time allowed: 50 minutes

Instructions:

- Calculators are not allowed.
- All electronic devices must be put away.
- Answers with insufficient or incorrect working will not receive full credit.
- Simplify answers whenever possible.

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	50	

1. (10 pts) Decide whether each statement is TRUE or FALSE. *No justification is required.*

- If A and B are square matrices of the same size, then $\det(A+B) = \det A + \det B$.

- If A and B are square matrices of the same size, then $\det(AB) = (\det A)(\det B)$.

- If A is an invertible matrix, then $\det A \neq 0$.

- The adjoint of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\text{adj}A = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$.

- If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$, then $\vec{u} + \vec{v} = \vec{v} + \vec{u}$.

- If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$, then $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \, \|\vec{v}\|$.

- If $\vec{a}$ and $\vec{b}$ are orthogonal, then $\text{proj}_{\vec{a}}(\text{proj}_{\vec{b}}(\vec{u})) = \vec{0}$.

- If $\vec{x}$ is a solution to the linear system $A\vec{x} = \vec{b}$, then $\vec{x}$ is orthogonal to every row of the matrix A.

- The equation of a plane can be determined from any three distinct points on the plane.

- If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^3$, then $\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2 \, \|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$.

2. (5 pts) Find the determinant of the matrix A using cofactor expansion down the first column.

$$A = \begin{bmatrix} 3 & -2 & 0 \\ 5 & 1 & -3 \\ 0 & -1 & 1 \end{bmatrix}$$

3. (5 pts) Find the determinant of the matrix B.

$$B = \begin{bmatrix} 0 & -4 & 4 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 2 & 0 & -1 \\ 3 & 0 & -2 & 2 \end{bmatrix}$$

4. (4 pts) Suppose that A is a 3×3 matrix with $\det A = 4$.

   (a) Find $\det(2A)$.

   (b) Find $\det B$, where B is obtained from A by replacing row 1 with the sum of row 1 and row 2.

   (c) Find $\det C$, where C is obtained from A by swapping row 1 and row 2.

   (d) Find $\det(A^T)$.

5. (6 pts) Let $\vec{u} = (2, 0, 1)$ and $\vec{v} = (-1, 2, 1)$. Compute each of the following, or explain why it is not possible.

   (a) $(\vec{u} \cdot \vec{v})\vec{v} =$

   (b) $\|\vec{u} + \vec{v}\| =$

   (c) $\vec{u} \times (\vec{u} \cdot \vec{v}) =$

6. (5 pts) Consider the two vectors $\vec{a} = (1, -1, -1, 1)$ and $\vec{b} = (2, -2, 0, -1)$. Find the component of $\vec{b}$ parallel to $\vec{a}$ and the component of $\vec{b}$ orthogonal to $\vec{a}$.

7. (5 pts) Conisder the vector $\vec{v} = (3, -2, -2)$ in $\mathbb{R}^3$ and the point $\vec{x}_0 = (3, 6, -1)$.
   (a) Find a scalar equation for the plane containing $\vec{x}_0$ and orthogonal to $\vec{v}$.

   (b) Find a vector equation for the line through $\vec{x}_0$ and parallel to $\vec{v}$.

8. (5 pts) Find the area of the triangle in $\mathbb{R}^3$ with vertices $A = (2, 0, 1)$, $B = (3, -2, 2)$, and $C = (1, 5, -1)$.

9. (5 pts) Suppose that $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$ with $\|\vec{u}\| = \|\vec{v}\|$. Show that $\vec{u} + \vec{v}$ is orthogonal to $\vec{u} - \vec{v}$.