Department of Mathematics – University of Tennessee

Math 251 Matrix Algebra I

Test 2 Practice

Name: ____________________

Time allowed: 50 minutes

Instructions:

- Calculators are not allowed.
- All electronic devices must be put away.
- Answers with insufficient or incorrect working will not receive full credit.
- Simplify answers whenever possible.

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	50	

1. (10 pts) Decide whether each statement is TRUE or FALSE. *No justification is required.*

   - The determinant of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$.

   - The determinant of an upper triangular matrix is the sum of the entries on the main diagonal.

   - If $\det A = 5$ then $\det(2A) = 10$.

   - If A is an $n \times n$ matrix, then $A\,(\mathrm{adj}A) = (\det A)\,I_n$.

   - If $\vec{u} + \vec{v} = \vec{u} + \vec{w}$, then $\vec{v} = \vec{w}$.

   - If $\vec{u} \cdot \vec{v} = 0$, then either $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

   - If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$, then $\|\vec{u} + \vec{v}\| \geq \|\vec{u}\| + \|\vec{v}\|$.

   - The vectors $(1, 2, 3)$ and $(-3, 2, -1)$ are orthogonal.

   - A vector equation for a line can be determined from any point on the line and any nonzero vector parallel to the line.

   - If $\vec{u}$ is any vector in $\mathbb{R}^3$, then $\vec{u} \times \vec{u} = \vec{0}$.

2. (5 pts) Find the determinant of the matrix A.

$$A = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 0 & 3 & 2 & 1 \\ 1 & 3 & -1 & 0 \\ 2 & -2 & 1 & 0 \end{bmatrix}$$

3. (5 pts) Use Cramer's Rule to find the value of y given the linear system below.

$$\begin{aligned} x + 3y - z &= 2 \\ 2x - 4y + \ z &= 0 \\ 2x + y - 8z &= 5 \end{aligned}$$

4. (10 pts) Let $\vec{u} = (2, -2, 1)$, $\vec{v} = (3, 3, 1)$, and $\vec{w} = (0, 4, -2)$. Compute each of the following, or explain why it is not possible.

   (a) $\vec{u} \cdot \vec{v} =$

   (b) $\|\vec{u}\| =$

   (c) $\text{proj}_{\vec{u}}\vec{v} =$

   (d) $\vec{u} + (\vec{v} \cdot \vec{w}) =$

   (e) $\vec{u} + (\vec{v} \times \vec{w}) =$

5. (5 pts) Find the component of $\vec{u} = (1, 2, 3, 4)$ that is parallel to $\vec{a} = (1, 1, 1, 1)$ and the component of $\vec{u}$ that is orthogonal to $\vec{a}$.

6. (5 pts) Conisder the plane in $\mathbb{R}^3$ that is parallel to the vector $\vec{v} = (1, 1, 1)$ and contains both of the points $\vec{x}_0 = (2, 6, -1)$ and $\vec{x}_1 = (-1, 4, 0)$.

   (a) Find a vector equation for the plane.

   (b) Find parametric equations for the plane.

7. (5 pts) Find the volume of the parallelepiped in $\mathbb{R}^3$ determined by the vectors $\vec{a} = (1, 0, 1)$, $\vec{b} = (1, 1, -1)$, and $\vec{c} = (0, -1, 1)$.

8. (5 pts) Let $\vec{u}$ and $\vec{v}$ be unit vectors in $\mathbb{R}^3$, and let $\vec{w} = \vec{u} + \vec{v}$. Show that the angle between $\vec{u}$ and $\vec{w}$ is equal to the angle between $\vec{v}$ and $\vec{w}$.

   Hint: it is enough to show that the cosines of the angles are equal.