Department of Mathematics – University of Tennessee

Math 251 Matrix Algebra I

Test 1

Name: ______________________________

Time allowed: 50 minutes

Instructions:

- Calculators are not allowed.
- All electronic devices must be put away.
- Answers with insufficient or incorrect working will not receive full credit.
- Simplify answers whenever possible.

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	50	

1. (10 pts) Decide whether each statement is TRUE or FALSE. *No justification is required.*

- A linear system with more variables than equations has no solutions.

- The matrix $\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \end{bmatrix}$ is in reduced row echelon form.

- If the products AB and BA are both defined, then A and B are square matrices of the same size.

- If A and B are invertible matrices of the same size, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

- The matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is an elementary matrix.

- If A is an $n \times n$ matrix that is not invertible, then the linear system $A\vec{x} = \vec{0}$ has infinitely many solutions.

- If A is row equivalent to the identity matrix I_n, then A is invertible.

- A matrix that is both upper triangular and symmetric is a diagonal matrix.

- Multiplication by the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ corresponds to a counterclockwise rotation by θ.

- If A is an $m \times n$ matrix, then the domain of the transformation T_A is $\mathbb{R}^m$.

2. (8 pts) Use the Gauss-Jordan elimination algorithm to solve the linear system.

$$\begin{aligned} x + 2y - 2z &= 3 \\ 2x + 5y - 6z &= 8 \\ x + y + z &= 2 \end{aligned}$$

3. (2 pts) Suppose that the augmented matrix for a linear system with variables x_1, x_2, x_3, x_4 has been reduced to the matrix below using elementary row operations. Write down the solution to the system.

$$\begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4. (10 pts) Let $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}$. Simplify each expression below or explain why it is not possible.

(a) $AA^T =$

(b) $B^2 =$

(c) $BA =$

(d) $\operatorname{tr}(B + B^T) =$

(e) $B^{-1} =$

5. (5 pts) How many solutions does each matrix equation have? Explain in complete sentences.

(a) $$\begin{bmatrix} 4 & 1 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 4 \\ 0 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & -5 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -1 \\ 0 \end{bmatrix}$$

(c) $$\begin{bmatrix} 2 & -2 & 0 \\ 0 & 5 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

6. (5 pts) Suppose that A is an $n \times n$ symmetric matrix.

(a) Show that A^2 is symmetric.

(b) Show that $A^2 - A + I$ is symmetric, where I is the $n \times n$ identity matrix.

7. (5 pts) Let $A \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

   (a) Find the inverse of A.

   (b) Use your answer to part (a) to solve the system $A\vec{x} = \vec{b}$.

8. (5 pts) The linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^3$ satisfies

$$T_A(1,0) = (2,1,1) \qquad \text{and} \qquad T_A(2,1) = (1,3,1).$$

   (a) Compute $T_A(0,1)$

   (b) What is the standard matrix A for this transformation?