Department of Mathematics – University of Tennessee

Math 251 Matrix Algebra I

Test 1 Practice

Name: ______________________________

Time allowed: 50 minutes

Instructions:

- Calculators are not allowed.
- All electronic devices must be put away.
- Answers with insufficient or incorrect working will not receive full credit.
- Simplify answers whenever possible.

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	50	

1. (10 pts) Decide whether each statement is TRUE or FALSE. *No justification is required.*

- Every homogeneous linear system is consistent.

- The matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ is in row echelon form.

- If A and B are square matrices of the same size, then $(A+B)^2 = A^2 + 2AB + B^2$.

- If AB is defined for matrices A and B, then $(AB)^T = B^T A^T$.

- If A and B are invertible matrices of the same size, then $A + B$ is invertible.

- Every elementary matrix is invertible.

- If A is invertible, and a multiple of the first row of A is added to the second row of A, then the resulting matrix is invertible.

- It is impossible for a system of linear equations to have exactly two solutions.

- The transpose of an upper triangular matrix is upper triangular.

- If $\vec{b}$ is a non-zero vector, then $T(\vec{x}) = \vec{x} + \vec{b}$ is a linear transformation.

2. (8 pts) Use the Gauss-Jordan elimination algorithm to solve the linear system.

$$\begin{aligned} 2x + 6y - 4z &= 2 \\ y + 2z &= 1 \\ x + 5y + 2z &= 3 \end{aligned}$$

3. (2 pts) Suppose that the augmented matrix for a linear system with variables x_1, x_2, x_3, x_4 has been reduced to the matrix below using elementary row operations. Write down the solution to the system.

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & -1 & 2 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

4. (10 pts) Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}$. Simplify each expression below or explain why it is not possible.

(a) $A + A^T =$

(b) $A^2 =$

(c) $BA =$

(d) $\mathrm{tr}(3B) =$

(e) $B^{-1} =$

5. (6 pts) Consider the matrix equation

$$\begin{bmatrix} 2 & -5 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & k^2-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -6 \\ k-1 \end{bmatrix}.$$

(a) For which value(s) of k are there an infinite number of solutions? Explain in complete sentences.

(b) For which value(s) of k is the system inconsistent? Explain in complete sentences.

(c) For which value(s) of k is there a unique solution? Explain in complete sentences.

6. (4 pts) Find the inverse of the matrix below.

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. (5 pts) Suppose that the matrix transformation T_A is defined by

$$T_A(x_1, x_2, x_3) = (2x_1 - x_3, 4x_1, x_1 + x_2 + x_3, 5x_3).$$

(a) What are the domain and codomain for the transformation?

(b) What is the standard matrix A for the transformation?

8. (5 pts) Suppose that the matrix A satisfies the equation $A^T A = A$.

(a) Prove that A is symmetric.

(b) Prove that $A^2 = A$.