Name: Solution.

1. Determine whether the matrices $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ are linearly independent in M_{22}.

   Recall that M_{22} is the set of all 2×2 matrices of real numbers.

$$k_1 \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} + k_2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Longrightarrow \begin{cases} k_1 + k_2 = 0 \\ 2k_2 + k_3 = 0 \\ k_1 + 2k_2 + 2k_3 = 0 \\ 2k_1 + k_2 + k_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 2 & 2 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - R_1 \\ R_4 \to R_4 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_4 \\ R_3 \to R_3 + R_4 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

From the second and third rows, we have $k_2 = 0$, $k_3 = 0$,

so from the first row $k_1 = 0$.

Thus the matrices are linearly independent.