**Name:** Solution

1. Use the Subspace Test to determine which of the sets are subspaces of $M_{22}$.

   *Recall that $M_{22}$ is the set of all $2 \times 2$ matrices of real numbers.*

   (a) All matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$.

Let $W$ be the set of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$.

$\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & 0 \\ 0 & b_1 + b_2 \end{bmatrix}$, so $W$ is closed under addition.

$k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix}$, so $W$ is closed under scalar multiplication.

Therefore, $W$ is a subspace of $M_{22}$.

   (b) All matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$.

Let $W$ be the set of all matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$.

$0\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, so $W$ is <u>not</u> closed under scalar multiplication.

Therefore, $W$ is <u>not</u> a subspace of $M_{22}$.

alternative counterexample:

$\begin{bmatrix} a_1 & 1 \\ 1 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 1 \\ 1 & b_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & 2 \\ 2 & b_1 + b_2 \end{bmatrix}$, so $W$ is <u>not</u> closed under addition.