Name: Solutions.

1. Consider the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for the null space of A.

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 4 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_2 = s$ and $x_3 = t$, so that $2x_1 - t = 0$ or $x_1 = \frac{t}{2}$.

The general solution is $\vec{x} = \left(\frac{t}{2}, s, t\right) = s(0,1,0) + t\left(\frac{1}{2}, 0, 1\right)$.

$S = \left\{ (0,1,0), \left(\frac{1}{2}, 0, 1\right) \right\}$ is a basis for the null space.

(b) Find a basis for the row space of A.

$$\begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$S = \left\{ \begin{bmatrix} 1 & 0 & -\frac{1}{2} \end{bmatrix} \right\}$ is a basis for the row space.