

Name:

1. Solve the given linear system by any method.

$$3x_1 + x_2 + x_3 + x_4 = 0$$

$$5x_1 - x_2 + x_3 - x_4 = 0$$

augmented matrix:

$$\begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix}$$

$$\downarrow R_1 \rightarrow \frac{1}{3}R_1$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix}$$

$$\downarrow R_2 \rightarrow R_2 - 5R_1$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{8}{3} & -\frac{2}{3} & -\frac{8}{3} & 0 \end{bmatrix}$$

$$\downarrow R_2 \rightarrow -\frac{3}{8}R_2$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{bmatrix}$$

$$\downarrow R_1 \rightarrow R_1 - \frac{1}{3}R_2$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{bmatrix}$$

Let $\underline{x_3 = s}$, $\underline{x_4 = t}$. Then:

$$\underline{x_1 = -\frac{1}{4}s}, \quad \underline{x_2 = -\frac{1}{4}s - t}.$$