Section 3.1: Vectors in 2-space, 3-space, and n-space

Objectives.

- Introduce the some terminology and notation for vectors.
- Understand vector operations in $\mathbb{R}^n$ geometrically and algebraically.
- Study some properties of vector operations.

A (geometric) vector is a quantity with a direction and a length, often represented by an arrow.

Two vectors can be added (geometrically) by placing the vectors end-to-end. (This is referred to as either the "triangle rule" or the "parallelogram rule".)

Multiplying a vector by a scalar changes ("scales") the length of the vector without changing the direction. If one vector is a scalar multiple of another, then we say the vectors are parallel. (Multiplying by a negative scalar reverses the orientation, but the result is still parallel to the original vector.)

We can view subtraction of a vector as "adding the negative of the vector".

If $P = (a_1, a_2, \ldots, a_n)$ and $Q = (b_1, b_2, \ldots, b_n)$ are two points in $\mathbb{R}^n$, then the vector from P to Q is

Two vectors $\vec{u} = (u_1, u_2, \ldots, u_n)$ and $\vec{v} = (v_1, v_2, \ldots, v_n)$ are equal if their components are equal. That is:

Example 1. Find the vector $\vec{u} = \overrightarrow{PQ}$ that has initial point $P = (3, -1)$ and terminal point $Q = (-2, 8)$.

Example 2. Find the initial point of a vector $\vec{w}$ that has terminal point $Q = (4, 7, 2)$ and is parallel to $\vec{v} = (-2, 1, 3)$ but has the opposite orientation.

Arithmetic with vectors (addition, subtraction, scalar multiplication) is done componentwise. If $\vec{u} = (u_1, u_2, \ldots, u_n)$ and $\vec{v} = (v_1, v_2, \ldots, v_n)$ are vectors in $\mathbb{R}^n$ and k is a scalar, then we define:

Example 3. Let $\vec{u} = (3, 1, 4, -2)$ and $\vec{v} = (1, -2, 3, 0)$. Simplify:

(a) $\vec{u} + \vec{v} =$

(b) $3\vec{u} - 4\vec{v} =$

Properties of vector operations. If $\vec{u}$, $\vec{v}$, and $\vec{w}$ are vectors in $\mathbb{R}^n$, and k and m are scalars, then:

1. $(\vec{u}+\vec{v})+\vec{w} =$
2. $\vec{u}+\vec{v} =$
3. $\vec{u}+\vec{0} =$
4. $\vec{u}+(-\vec{u}) =$
5. $k(\vec{u}+\vec{v}) =$
6. $(k+m)\vec{u} =$
7. $k(m\vec{u}) =$
8. $1\vec{u} =$

Proof of 2.

Example 4. Let $\vec{u} = (-1, 4, 6)$ and $\vec{v} = (3, 3, 3)$. Find the vector $\vec{x}$ satisfying $4\vec{x} - 2\vec{u} = 2\vec{x} - \vec{v}$.

Theorem. If $\vec{v}$ is a vector in $\mathbb{R}^n$ and k is a scalar, then

1. $0\vec{v} = \vec{0}$
2. $k\vec{0} = \vec{0}$
3. $(-1)\vec{v} = -\vec{v}$

Proof of 1.

A vector $\vec{w}$ in $\mathbb{R}^n$ is a linear combination of $\vec{v_1}, \vec{v_2}, \ldots, \vec{v_r} \in \mathbb{R}^n$ if

Example 5. Find scalars c_1, c_2, c_3 satisfying $c_1(1,2,2) + c_2(0,1,-1) + c_3(3,1,2) = (-1,7,7)$.

Example 6. Show that there is no choice of scalars a and b such that $a(3,-6) + b(-1,2) = (1,1)$.