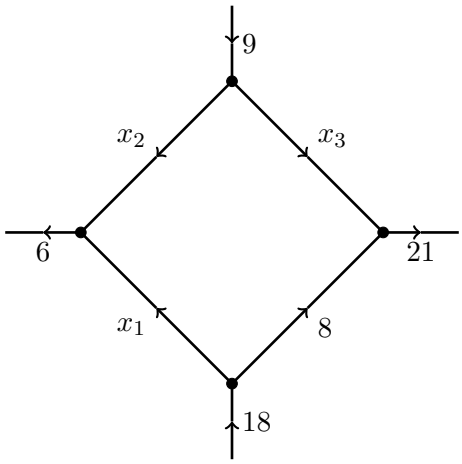
Section 1.10: Applications of Matrices

Network analysis

Problem 1. The diagram below shows a network consisting of four nodes with flow rates indicated on some of the branches. The flow rate (and direction of flow!) is unknown on the remaining three branches.

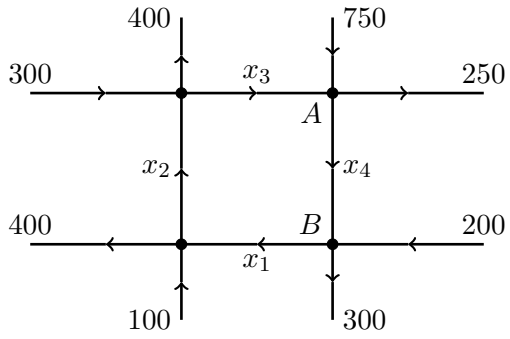


(a) Set up a system of linear equations that the flow rates x_1, x_2, and x_3 should satisfy.

(b) Solve the linear system.

(c) What does your answer for x_2 tell you about the diagram above?

Problem 2. The diagram below shows a network of one-way streets, with flow rates indicated on some of the streets (measured as the average number of vehicles per hour).



(a) Set up and solve a system of linear equations for the flow rates x_1, x_2, x_3, and x_4.

(b) The flow rate on the road from A to B has to be reduced because of construction. What is the minimum flow rate on this road that will keep traffic flowing on the other roads?

Balancing chemical equations.

Problem 3. The combustion of propane C_3H_8 in the presence of oxygen produces carbon dioxide and water.

$$a\,C_3H_8 \quad + \quad b\,O_2 \quad \rightarrow \quad c\,CO_2 \quad + \quad d\,H_2O$$

(a) Find a system of linear equations that the coefficients a, b, c, and d need to satisfy.

(b) Solve the system to find a balanced equation for the reaction.

(c) Repeat for the equation below (which represents photosynthesis).

$$a\,CO_2 \quad + \quad b\,H_2O \quad \rightarrow \quad c\,C_6H_{12}O_6 \quad + \quad d\,O_2$$

Polynomial interpolation.

Problem 4. Find the cubic polynomial $y = ax^3 + bx^2 + cx + d$ that passes through the points $(1, 3)$, $(2, -2)$, $(3, -5)$, and $(4, 0)$ using the following steps.

(a) Set up a system of linear equations in the coefficients a, b, c, and d.

(b) Solve the linear system.

(c) Check your answer by graphing your solution (using Desmos for example).