Section 1.1: Introduction to Systems of Linear Equations

Objectives.

- Identify linear and nonlinear equations, and systems of linear equations.
- Understand terminology related to linear systems and matrices.
- Solve simple linear systems and interpret their solutions geometrically.
- Introduce elementary row operations.

A linear equation in the variables $x_1, x_2, \ldots, x_n$ is an equation of the form

A homogeneous linear equation in the variables $x_1, x_2, \ldots, x_n$ is an equation of the form

Example 1. Underline the linear equations. Circle the homogeneous linear equations.

$$x + 4y = 9 \qquad w + 3x - y^2 + z = 3 \qquad -3x + 2y - \frac{1}{2}z = 0$$

$$x_1 - \sqrt{x_2} = 0 \qquad 4x_1 - 2x_2 + 3x_3 = 0 \qquad x_1 + x_2 + x_3 + x_4 = 1$$

A finite set of linear equations is called a system of linear equations (or linear system). The variables are called the unknowns.

A solution of a linear system is an assignment of a number to each unknown so that each equation in the linear system is true.

Example 2. Decide whether each set of numbers is a solution to the linear system below.

$$\begin{aligned} x + y + 3z &= 0 \\ 2x + y - z &= 5 \end{aligned}$$

(a) $x = 0,\ y = 0,\ z = 0$ (b) $x = 5,\ y = -5,\ z = 0$ (c) $x = 1,\ y = 2,\ z = -1$

The set of solutions of a linear equation in x and y is a line in the xy-plane, so a solution of a linear system in x and y corresponds to a point of intersection between lines.

Example 3. Solve each linear system, and interpret the solution(s) geometrically.

(a)
$$\begin{aligned} x + y &= 1 \\ 2x + y &= 4 \end{aligned}$$

(b)
$$\begin{aligned} x - 2y &= 3 \\ 2x - 4y &= 5 \end{aligned}$$

(c)
$$\begin{aligned} 3x + y &= 2 \\ 9x + 3y &= 6 \end{aligned}$$

The set of solutions of a linear equation in three variables is a plane, so a solution of a linear system in three variables corresponds to a point of intersection between planes.

Example 4. Solve the linear system and interpret the solution(s) geometrically.

$$\begin{aligned} x + y - z &= 4 \\ 2x + 2y - 2z &= 8 \\ 4x + 4y - 4z &= 16 \end{aligned}$$

More generally, a linear system is usually solved by performing <u>elementary row operations</u> on the <u>augmented matrix</u> for the system.

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

$$\begin{aligned} 2x \qquad - 4z &= -2 \\ z &= 2 \\ y \qquad &= 1 \end{aligned}$$

Elementary row operations.

1. Multiply a row by a nonzero constant.

2. Swap two rows.

3. Add a multiple of one row to another row.

Example 5. Solve the linear system and interpret the solution(s) geometrically.

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$