MATH 241 Calculus III
Chapter 15 Review
Multidimensional Integration

15.1 Double Integrals over Rectangular Regions

Evaluate the integral.

1) $\int_0^{4\pi} \int_0^{9\pi} (\sin x + \cos y)\, dx\, dy$

Evaluate the double integral over the given region.

2) $\iint_R xy\, dA, \quad R = \{(x, y): 9 \le x \le 10, 1 \le y \le 5\}$

Find the average value of the function f over the given region.

3) $f(x, y) = 10x + 3y; R = \{(x, y): 0 \le x \le 2, 0 \le y \le 10\}$

Determine whether the statement is true or false.

4) $\int_{-1}^{2} \int_0^6 x^2 \sin(x - y)\, dx dy = \int_0^6 \int_{-1}^{2} x^2 \sin(x - y)\, dy dx$

5) $\int_1^2 \int_3^4 x^2 e^y\, dx dy = \int_3^4 x^2\, dx \int_1^2 e^y\, dy$

6) What is the integral of the constant function f(x,y) = 5 over the rectangle [-2,3] x [2,4]?

15.2 Double Integrals over General Regions

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

7) The parabola $y = x^2$ and the line $y = 7x + 18$

Evaluate the integral.

8) $\int_0^{\ln 6} \int_{e^y}^6 e^y\, dx\, dy$

Write an equivalent double integral with the order of integration reversed.

9) $\int_0^4 \int_{7x/4 + 2}^9 dy\, dx$

Determine whether the statement is true or false.

10) $\int_0^1 \int_0^x \sqrt{x+y}\, dxdy = \int_0^x \int_0^1 \sqrt{x+y}\, dydx$

15.3 Double Integrals in Polar Coordinates

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

11) $\int_0^7 \int_0^{\sqrt{49-y^2}} (x^2+y^2)\, dx\, dy$

Find the area of the region specified in polar coordinates.

12) the region enclosed by the curve $r = 5 + \cos\theta$

15.4 Double Integral Applications

Find the center of mass of a thin plate covering the given region with the given density function.

13) The triangular region cut from the first quadrant by the line $y = -x + 4$, with density $\varrho(x) = x$

Solve the problem.

14) Find the moment of inertia about the x-axis of a thin plane of constant density $\varrho = 10$ bounded by the coordinate axes and the line $\frac{x}{2} + \frac{y}{8} = 1$.

15.6 Triple Integrals

Solve the problem.

15) Write an iterated triple integral in the order dz dy dx for the volume of the rectangular solid in the first octant bounded by the planes $x = 9$, $y = 5$, and $z = 6$.

Evaluate the integral.

16) $\int_0^2 \int_2^{y^2} \int_2^z yz\, dx\, dz\, dy$

15.7 Triple Integrals in Cylindrical Coordinates

Evaluate the cylindrical coordinate integral.

17) $\int_0^{8\pi} \int_0^2 \int_r^{4r} dz\, r\, dr\, d\theta$

Solve the problem.

18) Let D be the region bounded below by the xy-plane, above by the sphere $x^2 + y^2 + z^2 = 49$, and on the sides by the cylinder $x^2 + y^2 = 36$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration dz dr $d\theta$.

Set up the iterated integral for evaluating $\iiint_D f(r, \theta, z)\, dz\, r\, dr\, d\theta$ over the given region D.

19) D is the right circular cylinder whose base is the circle $r = 8 \sin \theta$ in the xy-plane and whose top lies in the plane $z = 10 - y$.

Determine whether the statement is true or false.

20) The integral $\iiint_E \varrho r^3\, dz\, dr\, d\theta$ represents the moment of inertia about the z-axis of a solid E with constant denstiy ϱ.

21) The integral $\int_0^{2\pi} \int_0^2 \int_r^2 dz\, dr\, d\theta$ represents the volume enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$.

15.8 Triple Integrals in Spherical Coordinates

Evaluate the spherical coordinate integral.

22) $\int_0^{\pi} \int_0^{\pi} \int_0^{10 \sin \varphi} \varrho^2 \sin \varphi\, d\varrho\, d\varphi\, d\theta$

Solve the problem.

23) Let D be the region that is bounded below by the cone $\varphi = \frac{\pi}{4}$ and above by the sphere $\varrho = 10$. Set up the triple integral for the volume of D in spherical coordinates.

Answer Key
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1) 8π
2) 114
3) 25
4) True
5) True
6) 50
7) $\int_{-2}^{9} \int_{x^2}^{7x+18} dy\, dx$
8) $\frac{25}{2}$
9) $\int_{2}^{9} \int_{0}^{4(y-2)/7} dx\, dy$
10) False
11) $\frac{2401\pi}{8}$
12) $\frac{51}{2}\pi$
13) $\bar{x} = 2, \bar{y} = 1$
14) $\frac{2560}{3}$
15) $\int_{0}^{9} \int_{0}^{5} \int_{0}^{6} dz\, dy\, dx$
16) $\frac{8}{3}$
17) 64π
18) $\int_{0}^{2\pi} \int_{0}^{6} \int_{0}^{\sqrt{49-r^2}} r\, dz\, dr\, d\theta$
19) $\int_{0}^{\pi} \int_{0}^{8\sin\theta} \int_{0}^{10-r\sin\theta} f(r, \theta, z)\, dz\, r\, dr\, d\theta$
20) True
21) False
22) $125\pi^2$
23) $\int_{0}^{2\pi} \int_{0}^{\pi/4} \int_{0}^{10} \varrho^2 \sin\varphi\, d\varrho\, d\varphi\, d\theta$