

MATH 241 Calculus III
Chapter 13 Review
Vector Functions

13.1 Vector Functions and Space Curves

Find a function $r(t)$ that describes the line or line segment.

- 1) The line through $P(4, 9, 3)$ and $Q(1, 6, 7)$

Find the domain of the function.

2) $r(t) = \sqrt{t-1}\mathbf{i} + \ln(t-2)\mathbf{j} + \frac{1}{t-5}\mathbf{k}$

Evaluate the limit.

3) $\lim_{t \rightarrow \ln 6} = (6e^{-t}\mathbf{i} + 3e^{-t}\mathbf{j})$

13.2 Derivatives and Integrals of Vector Functions

Differentiate the function.

4) $r(t) = (8t^2 - 12)\mathbf{i} + \left(\frac{1}{18}t^3\right)\mathbf{j}$

Find the unit tangent vector of the given curve.

5) $r(t) = \left(3 \cos \frac{5}{3}t\right)\mathbf{i} + \left(3 \sin \frac{5}{3}t\right)\mathbf{j} - 12t\mathbf{k}$

For the smooth curve $r(t)$, find the parametric equations for the line that is tangent to r at the given parameter value $t = t_0$.

6) $r(t) = (4t^2 - 3t)\mathbf{i} + (t + 7)\mathbf{j} + \mathbf{k}$; $t_0 = 2$

Compute $r''(t)$.

7) $r(t) = (7 \ln(5t))\mathbf{i} + (3t^3)\mathbf{j}$

Solve the initial value problem.

8) Differential Equation: $\frac{dr}{dt} = \mathbf{i} + (4t^3 - 10t)\mathbf{j} + \frac{1}{t+2}\mathbf{k}$

Initial Condition: $r(0) = 2\mathbf{j} + (\ln 2)\mathbf{k}$

Evaluate the integral.

9) $\int_0^{\pi/4} [(2\sec^2 t)\mathbf{i} - (7 + \sin t)\mathbf{j} - (3\sec t \tan t)\mathbf{k}]dt$

Answer Key

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1) $\mathbf{r}(t) = \langle 4 - 3t, 9 - 3t, 3 + 4t \rangle$

2) $(2, 5) \cup (5, \infty)$

3) $\mathbf{i} + \frac{1}{2}\mathbf{j}$

4) $\mathbf{r}'(t) = (16t)\mathbf{i} + \left(\frac{1}{6}t^2\right)\mathbf{j}$

5) $\mathbf{T} = \left(-\frac{5}{13} \sin \frac{5}{3}t\right)\mathbf{i} + \left(\frac{5}{13} \cos \frac{5}{3}t\right)\mathbf{j} - \frac{12}{13}\mathbf{k}$

6) $x = 10 + 13t, y = 9 + t, z = 1$

7) $\mathbf{r}''(t) = -\frac{7}{t^2}\mathbf{i} + 18t\mathbf{j}$

8) $\mathbf{r}(t) = t\mathbf{i} + \left(\frac{t^2}{4}(4t^2 - 20) + 2\right)\mathbf{j} + \ln(t + 2)\mathbf{k}$

9) $2\mathbf{i} + \left(\frac{2\sqrt{2} - 7\pi - 4}{4}\right)\mathbf{j} + 3(1 - \sqrt{2})\mathbf{k}$