MATH 241 Calculus III
Chapter 12 Review
Vectors and Geometry of Space

12.1 Three-Dimensional Coordinate Systems

Find the distance between points P_1 and P_2.

1) $P_1(7, 8, -7)$ and $P_2(9, 5, -13)$

Find the center and radius of the sphere.

2) $x^2 + y^2 + z^2 - 8x - 2y + 8z = 3$

Find an equation for the sphere with the given center and radius.

3) Center $(0, -8, -1)$, radius = 10

12.2 Vectors

Find the indicated vector.

4) Let $u = \langle -1, -8 \rangle$, $v = \langle 4, -1 \rangle$. Find $-5u + 4v$.

Find the magnitude.

5) Let $u = \langle -1, 2 \rangle$. Find the magnitude (length) of the vector: $7u$.

Find the component form of the specified vector.

6) The vector $\overrightarrow{PQ}$, where $P = (-10, -10)$ and $Q = (-2, -1)$

Express the vector in the form $v = v_1 i + v_2 j + v_3 k$.

7) $\overrightarrow{P_1P_2}$ if P_1 is the point $(-6, -3, 4)$ and P_2 is the point $(-4, -6, 0)$

12.3 Dot Product

Find $v \cdot u$.

8) $v = -4i + 9j$ and $u = 6i + 5j$

Find the vector $\text{proj}_v u$.

9) $v = 3i - j + 3k$, $u = 10i + 11j + 2k$

Solve the problem.

10) How much work does it take to slide a box 12 meters along the ground by pulling it with a 180 N force at an angle of 45° from the horizontal?

11) The unit vectors u and v are combined to produce two new vectors a = u + v and b = u - v. Show that a and b are orthogonal. Assume $u \neq v$.

12.4 Cross Product

Find the length and direction (when defined) of $u \times v$.

12) $u = 4i + 2j + 8k$, $v = -i - 2j - 2k$

Solve the problem.

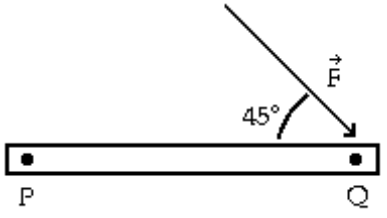
13) Find the area of the parallelogram determined by the points P(7,-5,5), Q(1,-5,2), R(8,3,3) and S(2,3,0).

Find the triple scalar product $(u \times v) \cdot w$ of the given vectors.

14) $u = i + j + k$; $v = 9i + 7j + 2k$; $w = 10i + 6j + 5k$

Solve the problem.

15) Find the magnitude of the torque in foot-pounds at point P for the following lever:



$|\overrightarrow{PQ}| = 5$ in. and $|F| = 20$ lb

Determine whether the following is always true or not always true. Given reasons for your answers.

16) $|u| = \sqrt{u \cdot u}$

17) $u \times 0 = 0$

18) $u \times (v + w) = u \times v + u \times w$

19) $(u \times v) \cdot w = u \cdot (w \times v)$

20) $u \times v = -(v \times u)$

21) $(u \times v) \cdot v = 0$

22) $c(u \cdot v) = cu \cdot cv$ (any number c)

12.5 Equations of Lines and Planes

Find parametric equations for the line described below.

23) The line through the point P(3, 3, 5) parallel to the vector $-4i + 3j - 7k$

Find a parametrization for the line segment joining the points.

24) (4, 0, 3), (0, 4, 0)

Write the equation for the plane.

25) The plane through the point P(-2, -5, 6) and normal to $n = -6i - 3j + 5k$.

Calculate the requested distance.

26) The distance from the point S(4, -6, 8) to the plane $2x + 2y + z = -6$

Find the intersection.

27) $x = -2 + 2t, y = 1 + 10t, z = -2 + 7t$; $-10x + 2y + 8z = 4$

Answer Key
Testname:

1) 7
2) C(4, 1, -4), a = 6
3) $x^2 + y^2 + z^2 + 16y + 2z = 35$
4) $\langle 21, 36 \rangle$
5) $7\sqrt{5}$
6) $\langle 8, 9 \rangle$
7) $v = 2i - 3j - 4k$
8) 21
9) $\frac{75}{19}i - \frac{25}{19}j + \frac{75}{19}k$
10) $1080\sqrt{2}$ joules
11) $u = u_x\, i + u_y j$ and $v = v_x\, i + v_y j$, so
$a = u + v = (u_x + v_x)\, i + (u_y + v_y)j$ and $b = u - v = (u_x - v_x)\, i + (u_y - v_y)j$

Take the dot product $a \cdot b$:
$a \cdot b = (u + v) \cdot (u - v) = (u_x + v_x)(u_x - v_x) + (u_y + v_y)(u_y - v_y)$

$= u_x^2 - v_x^2 + u_y^2 - v_y^2 = (u_x^2 + u_y^2) - (v_x^2 + v_y^2)$

$= |u| - |v| = 1 - 1 = 0$

Since the dot product of the two non-zero vectors is zero they are orthogonal.

12) $6\sqrt{5}; \frac{2\sqrt{5}}{5}i - \frac{\sqrt{5}}{5}k$
13) $3\sqrt{345}$
14) -18
15) $\frac{25}{6}\sqrt{2}$ ft-lb
16) Always true by definition
17) Always true by definition of 0
18) Always true by distributive property
19) Not always true; $(u \times v) \cdot w = u \cdot (v \times w)$, but $v \times w = -(w \times v)$ from which it follows that the original equation false if $w \times v \neq 0$.
20) Always true by definition of the cross product
21) Always true because $u \times v$ and v are orthogonal
22) Not always true; The statement if false if $c \neq 0,1$.
23) $x = -4t + 3, y = 3t + 3, z = -7t + 5$
24) $x = -4t + 4, y = 4t, z = -3t + 3, 0 \leq t \leq 1$
25) $-6x - 3y + 5z = 57$
26) $\frac{10}{3}$
27) $\left(-\frac{29}{14}, \frac{9}{14}, -\frac{9}{4} \right)$