

Calculus III
Math 241-002
Fall 2025
Quiz 12

Name Solution

Find the area of the surface S that is part of the plane $\mathbf{r}(u, v) = \langle u + v, 2 - 3u, 1 + u - v \rangle$ given by $0 \leq u \leq 2, -1 \leq v \leq 1$.

(a.) Write the expression for the integrand below.

$$A(S) = \iint_S dS = \iint_D |\vec{r}_u \times \vec{r}_v| dA$$

(b.) Calculate the value of the integrand. Show all necessary work.

$$\vec{r}_u = \langle 1, -3, 1 \rangle, \quad \vec{r}_v = \langle 1, 0, -1 \rangle$$

$$\begin{aligned} \vec{r}_u \times \vec{r}_v &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i}(3-0) - \hat{j}(-1-1) + \hat{k}(0-(-3)) \\ &= \langle 3, 2, 3 \rangle \end{aligned}$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{9+4+9} = \sqrt{22}$$

(c.) Convert the integral to an appropriate iterated integral and evaluate it.

$$A(S) = \iint_S dS = \int_{-1}^1 \int_0^2 \sqrt{22} \, du \, dv = \sqrt{22} (2-0)(1-(-1)) = 4\sqrt{22}$$