Calculus III
Math 241-002
Fall 2025
Quiz 11

Name Solution

Let $\mathbf{F}(x, y) = 2xe^{-y}\mathbf{i} + (2y - x^2e^{-y})\mathbf{j}$, $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, and C a path from (1, 0) to (2, 1).

(a.) Show that $\mathbf{F}$ is a conservative vector field.

$\vec{F}$ is defined on $\mathbb{R}^2$, an open, simply-connected region.

$$\frac{\partial(2xe^{-y})}{\partial y} = -2xe^{-y} = \frac{\partial(2y - x^2e^{-y})}{\partial x}$$

(b.) Find a function $f(x, y)$ such that $\nabla f = \mathbf{F}$.

$$\frac{\partial f}{\partial x} = 2xe^{-y} \Rightarrow f(x,y) = x^2e^{-y} + g(y)$$

$$\frac{\partial f}{\partial y} = -x^2e^{-y} + g'(y) = 2y - x^2e^{-y} \Rightarrow g'(y) = 2y \Rightarrow$$

$g(y) = y^2 + K$. Let $K = 0$.

$$f(x,y) = x^2e^{-y} + y^2$$

(c.) Evaluate the integral.

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(2,1) - f(1,0) = 2^2e^{-1} + 1^2 - (1e^0 + 0)$$

$$= \frac{4}{e}$$