

Calculus III
Math 241-002
Fall 2025
Quiz 10

Name Solution

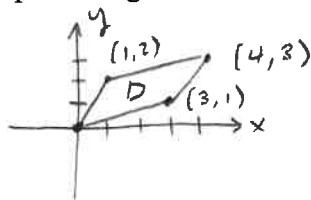
Evaluate $\iint_D y \, dA$ where D is a parallelogram with vertices $(0, 0)$, $(3, 1)$, $(4, 3)$, and $(1, 2)$, using the transformation $x = u + 3v$, $y = 2u + v$.

(a.) Find $u = g(x, y)$ and $v = h(x, y)$.

$$u = x - 3v = x - 3(y - 2u) = x - 3y + 6u \Rightarrow u = -\frac{1}{5}x + \frac{3}{5}y$$

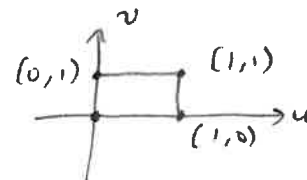
$$v = y - 2u = y - 2\left(-\frac{1}{5}x + \frac{3}{5}y\right) = y + \frac{2}{5}x - \frac{6}{5}y \Rightarrow v = \frac{2}{5}x - \frac{1}{5}y$$

(b.) Graph the region D in the x - y plane and the transformation of D in the u - v plane.



$$u = -\frac{1}{5}x + \frac{3}{5}y$$

$$v = \frac{2}{5}x - \frac{1}{5}y$$



(c.) Calculate the Jacobian.

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1 - 6 = -5$$

(d.) Evaluate the integral.

$$\begin{aligned} \iint_D y \, dA &= \int_0^1 \int_0^1 (2u + v) |-5| \, du \, dv = 5 \int_0^1 \left[u^2 + uv \right]_{u=0}^{u=1} dv \\ &= 5 \int_0^1 (1 + v) \, dv = 5 \left[v + \frac{v^2}{2} \right]_0^1 = 5 \left(1 + \frac{1}{2} \right) = \frac{15}{2} \end{aligned}$$