

Calculus III
Math 241-002
Fall 2025
Quiz 9

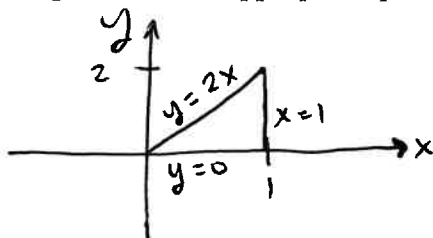
Name Solution

Evaluate $\iiint_E y \, dV$, where E is a solid region lying below the plane $z = 3 - x - y$ and above the triangular region with vertices $(0, 0, 0)$, $(1, 0, 0)$, and $(1, 2, 0)$, using the following steps:

- (a.) Regarding E as a Type I region, fill in the missing parts of the triple integral below and then integrate one iteration to get a double integral over a region D .

$$\iiint_E y \, dV = \iint_D \int_0^{3-x-y} y \, dz \, dA = \iint_D y z \Big|_0^{3-x-y} dA = \iint_D (3y - xy - y^2) \, dA$$

- (b.) Draw the region D in the appropriate plane and write the equations of the boundaries.



- (c.) Fill in the missing parts of the double and iterated integrals below and evaluate.

$$\begin{aligned} \iiint_E y \, dV &= \iint_D (3y - xy - y^2) \, dA = \int_0^1 \int_0^{2x} (3y - xy - y^2) \, dy \, dx \\ &= \int_0^1 \left[\frac{3}{2} y^2 - \frac{xy^2}{2} - \frac{y^3}{3} \right]_0^{2x} dx \\ &= \int_0^1 \left(6x^2 - 2x^3 - \frac{8}{3} x^3 \right) dx = 2x^3 - \frac{1}{2} x^4 - \frac{2}{3} x^4 \Big|_0^1 \\ &= 2 - \frac{1}{2} - \frac{2}{3} = \frac{12}{6} - \frac{3}{6} - \frac{4}{6} = \frac{5}{6} \end{aligned}$$