

Calculus III
Math 241-005
Fall 2025
Quiz 7

Name Solution

Find the local maximum and minimum values and saddle points of the function

$$f(x, y) = x^3 + y^3 + 3xy$$

$$\left. \begin{aligned} f_x = 3x^2 + 3y = 0 &\Rightarrow y = -x^2 \\ f_y = 3y^2 + 3x = 0 &\Rightarrow y^2 + x = 0 \end{aligned} \right\} \Rightarrow x^4 + x = x(x^3 + 1) = 0 \Rightarrow$$

$x = 0, -1$. The critical points are $(0, 0)$ and $(-1, -1)$.

$$D(x, y) = f_{xx}f_{yy} - f_{xy}^2 = (6x)(6y) - 3^2 = 36xy - 9$$

$$D(0, 0) = -9 < 0 \Rightarrow (0, 0) \text{ is a saddle point}$$

$$D(-1, -1) = 36 - 9 = 27 > 0 \text{ and } f_{xx}(-1, -1) = -6 < 0 \Rightarrow$$

$$f(-1, -1) = 1 \text{ is a local maximum.}$$