

Calculus III
Math 241-002
Fall 2025
Quiz 6

Name Solution

For $yz + x \ln y = z^2 + 2$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ using any technique. Simplify your answer.

Let $F(x, y, z) = yz + x \ln y - z^2 = 2$. Then

$$F_x = \ln y$$

$$F_y = z + \frac{x}{y}$$

$$F_z = y - 2z$$

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} = - \frac{\ln y}{y - 2z} = \frac{\ln y}{2z - y}$$

$$\frac{\partial z}{\partial y} = - \frac{F_y}{F_z} = - \frac{z + \frac{x}{y}}{y - 2z} = \frac{x + yz}{2yz - y^2}$$

Alternatively,

$$\frac{\partial}{\partial x} (yz + x \ln y) = \frac{\partial}{\partial x} (z^2 + 2) \Rightarrow y \frac{\partial z}{\partial x} + \ln y = 2z \frac{\partial z}{\partial x} \Rightarrow$$

$$\frac{\partial z}{\partial x} = \frac{\ln y}{2z - y}$$

$$\frac{\partial}{\partial y} (yz + x \ln y) = \frac{\partial}{\partial y} (z^2 + 2) \Rightarrow z + y \frac{\partial z}{\partial y} + \frac{x}{y} = 2z \frac{\partial z}{\partial y} \Rightarrow$$

$$y^2 \frac{\partial z}{\partial y} - 2yz \frac{\partial z}{\partial y} = -x - yz \Rightarrow \frac{\partial z}{\partial y} = \frac{x + yz}{2yz - y^2}$$