

Calculus III
Math 241-002
Fall 2025
Quiz 5

Name Solution

For $f(x, y) = 2x^2y^3 + x \sin y$, find $\frac{\partial f}{\partial x}$ using the limit definition of the partial derivative.

$$\frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(x+h)^2y^3 + (x+h)\sin y - (2x^2y^3 + x \sin y)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x^2y^3 + 4xhy^3 + 2h^2y^3 + x \sin y + h \sin y - 2x^2y^3 - x \sin y}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4xhy^3 + 2h^2y^3 + h \sin y}{h}$$

$$= \lim_{h \rightarrow 0} (4xy^3 + 2hy^3 + \sin y)$$

$$= 4xy^3 + \sin y$$