

Calculus III  
Math 241-002  
Fall 2025  
Quiz 4

Name Solution

Find the curvature of  $\mathbf{r}(t) = \langle t, \frac{1}{2}t^2, t^2 \rangle$  at the point  $(1, \frac{1}{2}, 1)$ .

Method 1:  $\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$

$$\vec{r}'(t) = \langle 1, t, 2t \rangle$$

$$|\vec{r}'(t)| = \sqrt{1 + t^2 + 4t^2} = \sqrt{1 + 5t^2}$$

$$\hat{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \langle 1, t, 2t \rangle (1 + 5t^2)^{-1/2}$$

$$\vec{T}'(t) = \langle 0, 1, 2 \rangle (1 + 5t^2)^{-1/2} - \frac{1}{2}(1 + 5t^2)^{-3/2}(10t) \langle 1, t, 2t \rangle$$

At  $(1, \frac{1}{2}, 1)$ ,  $t = 1$ .

$$\vec{T}'(1) = \frac{\langle 0, 1, 2 \rangle}{\sqrt{6}} - \frac{5}{6\sqrt{6}} \langle 1, 1, 2 \rangle$$

$$= \frac{\langle 0, 6, 12 \rangle - \langle 5, 5, 10 \rangle}{6\sqrt{6}}$$

$$= \frac{\langle -5, 1, 2 \rangle}{6\sqrt{6}}$$

$$|\vec{T}'(1)| = \frac{\sqrt{25 + 1 + 4}}{6\sqrt{6}} = \frac{\sqrt{5}}{6}$$

$$|\vec{r}'(1)| = \sqrt{6}$$

$$\kappa(1) = \frac{\sqrt{5}/6}{\sqrt{6}} = \frac{1}{6} \sqrt{\frac{5}{6}}$$

Method 2:  $\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$

$$\vec{r}'(t) = \langle 1, t, 2t \rangle$$

$$\vec{r}''(t) = \langle 0, 1, 2 \rangle$$

At  $(1, \frac{1}{2}, 1)$ ,  $t = 1$ .

$$\vec{r}'(1) = \langle 1, 1, 2 \rangle$$

$$\vec{r}''(1) = \langle 0, 1, 2 \rangle$$

$$\vec{r}'(1) \times \vec{r}''(1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(2-2) - \hat{j}(2-0) + \hat{k}(1-0)$$

$$= \langle 0, -2, 1 \rangle$$

$$|\vec{r}'(1) \times \vec{r}''(1)| = \sqrt{4+1} = \sqrt{5}$$

$$|\vec{r}'(1)| = \sqrt{1+1+4} = \sqrt{6}$$

$$\kappa(1) = \frac{\sqrt{5}}{\sqrt{6}^3} = \frac{1}{6} \sqrt{\frac{5}{6}}$$