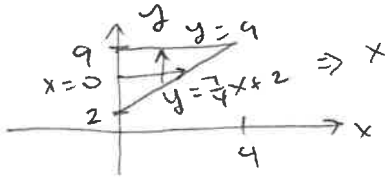
Mon 11/3

Test III Review

Office Hours: 5:30 - 6:30 Mon
3:00 - 6:00 Tues

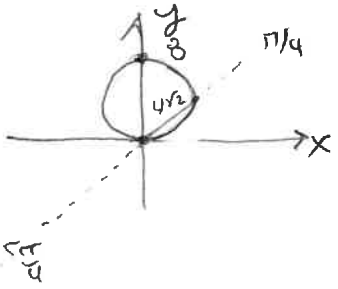
(9) $\int_0^4 \int_{\frac{7}{4}x+2}^{9} dy\,dx$ Write an equivalent integral reversing the order



$\Rightarrow x = \frac{4}{7}(y-2)$ $\quad y = \frac{7}{4}x + 2$

$$\int_2^9 \int_0^{\frac{4}{7}(y-2)} dx\,dy$$

(19) $r = 8\sin\theta$



$r(\pi) = 8\sin\pi = 0$

$r\left(\frac{5\pi}{4}\right) = 8\sin\frac{5\pi}{4} = -4\sqrt{2}$

$(r,\theta) = \left(-4\sqrt{2}, \frac{5\pi}{4}\right) = \left(4\sqrt{2}, \frac{\pi}{4}\right)$

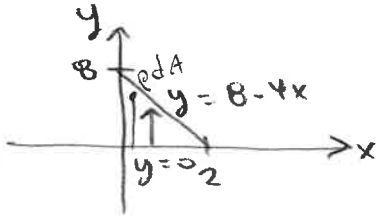
$0 \le \theta \le \pi$

$0 \le r \le 8\sin\theta$

$0 \le z \le 10 - y = 10 - r\sin\theta$

$$\int_0^{\pi} \int_0^{8\sin\theta} \int_0^{10-r\sin\theta} \delta(r,\theta,z)\, r\, dz\, dr\, d\theta$$

(14) $I_x = \iint_D \rho y^2\, dA$



$\rho = 10$

$\frac{x}{2} + \frac{y}{8} = 1 \Rightarrow y = 8 - 4x$

$$I_x = \int_0^2 \int_0^{8-4x} 10y^2\, dy\, dx = 10\int_0^2 \frac{y^3}{3}\Big|_0^{8-4x} dx$$

$$= \frac{10}{3}\int_0^2 (8-4x)^3\, dx = \frac{10}{3}\int_0^2 \frac{1}{-4}(8-4x)^3\, d(8-4x)$$

$$= \frac{-10}{12}\frac{(8-4x)^4}{4}\Big|_0^2 = \frac{-5}{24}\left(-8^4\right) = \frac{40}{3}(64) = \frac{2560}{3}$$

(22) $\int_0^{\pi}\int_0^{\pi}\int_0^{10\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \int_0^{\pi}\int_0^{\pi} \frac{\rho^3}{3}\sin\phi \Big|_0^{10\sin\phi} d\phi \, d\theta$

$= \int_0^{\pi}\int_0^{\pi} \frac{1000}{3}\sin^3\phi \sin\phi \, d\phi \, d\theta$

$= \frac{1000}{3}\int_0^{\pi}\int_0^{\pi} \sin^4\phi \, d\phi \, d\theta = \frac{1000}{3}\int_0^{\pi} \frac{3\pi}{8} d\theta = \frac{1000\pi}{8}\theta\Big|_0^{\pi} = 125\pi^2$

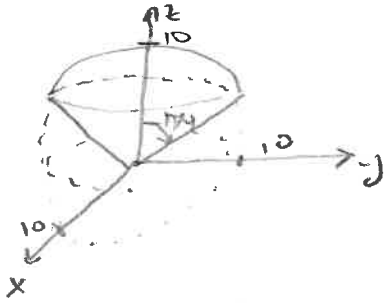
where $\int_0^{\pi} \sin^4\phi \, d\phi = \int_0^{\pi} (\sin^2\phi)^2 d\phi = \int_0^{\pi} \left(\frac{1}{2} - \frac{\cos 2\phi}{2}\right)^2 d\phi$

$= \int_0^{\pi} \left(\frac{1}{4} - \frac{\cos 2\phi}{2} + \frac{\cos^2 2\phi}{4}\right) d\phi$

$= \int_0^{\pi} \left(\frac{1}{4} - \frac{\cos 2\phi}{2} + \frac{1}{8} + \frac{\cos 4\phi}{8}\right) d\phi$

$= \frac{3}{8}\phi - \frac{\sin 2\phi}{4} + \frac{\sin 4\phi}{32}\Big|_0^{\pi} = \frac{3}{8}\pi.$

(23)



$0 \le \rho \le 10$
$0 \le \phi \le \frac{\pi}{4}$
$0 \le \theta \le 2\pi$

$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{10} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$