Mon 10/13

§14.7 Maximum and Minimum Values

- Quiz 7 Tuesday 10/14 covers local extrema and saddle points.
- Absolute extrema
- Optimization Problem

HW 14.7 due Monday 10/20

Recall that for $f(x,y)$, the determinant is

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - f_{xy}^2$$

three cases:

1) $D(a,b) > 0$ and $f_{xx}(a,b) > 0 \Rightarrow f(a,b)$ is a local min

2) $D(a,b) > 0$ and $f_{xx}(a,b) < 0 \Rightarrow f(a,b)$ is a local max

3. $D(a,b) < 0 \Rightarrow (a,b)$ is a saddle point

Example: Find the local extrema and saddle points for $f(x,y) = 4 + x^3 + y^3 - 3xy$

Solution: $f_x = 3x^2 - 3y = 0 \Rightarrow x^2 - y = 0 \Rightarrow y = x^2$

$f_y = 3y^2 - 3x = 0 \qquad (x^2)^2 - x = 0 \Rightarrow x^4 - x = 0$

$$\left.\begin{array}{l} f_{xx} = 6x \\ f_{yy} = 6y \\ f_{xy} = -3 \end{array}\right\} \Rightarrow D = 36xy - (-3)^2 = 36xy - 9$$

$x^4 - x = x(x^3 - 1) = 0 \Rightarrow x = 0$ or $x^3 - 1 = 0 \Rightarrow x = 1$

Critical points are $(0,0)$ and $(1,1)$.

$D(0,0) = -9 < 0 \Rightarrow (0,0)$ is a saddle point

$D(1,1) = 36 - 9 > 0$ and $f_{xx}(1,1) = 6 > 0 \Rightarrow$

$f(1,1) = 4 + 1 + 1 - 3 = 3$ is a local min

Example: Find the local extrema and saddle points for $f(x,y) = x^3 - 3x + 3xy^2$

Solution: $f_x = 3x^2 - 3 + 3y^2$

$f_y = 6xy$

$$\left.\begin{array}{l} f_{xx} = 6x \\ f_{yy} = 6x \\ f_{xy} = 6y \end{array}\right\} D = 36x^2 - 36y^2 = 36(x^2 - y^2)$$

$3x^2 - 3 + 3y^2 = 0 \Rightarrow x^2 + y^2 = 1$

$6xy = 0 \Rightarrow x = 0$ or $y = 0$

Critical points are $(0, \pm 1)$, $(\pm 1, 0)$

$D(0,1) = -36 < 0 \Rightarrow (0,1)$ is a saddle point

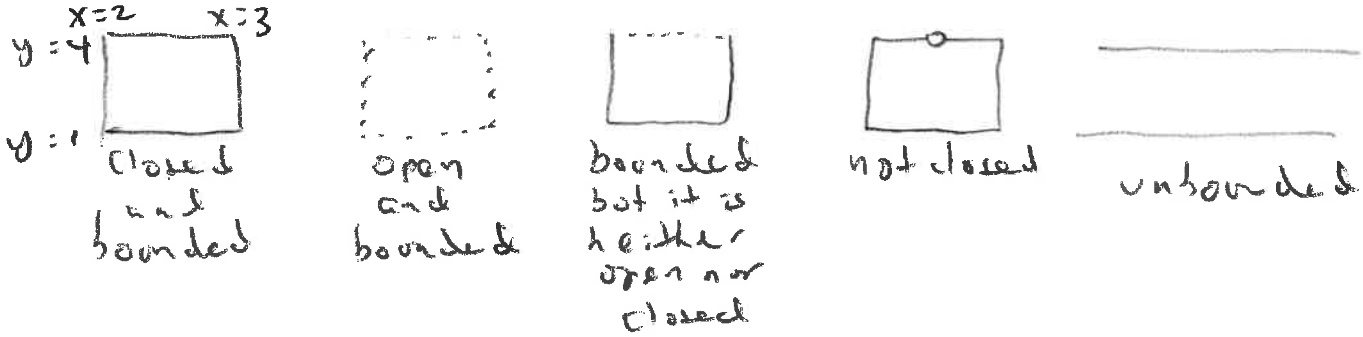
$D(0,-1) = -36 < 0 \Rightarrow (0,-1)$ is a saddle point

$D(1,0) = 36 > 0$ and $f_{xx}(1,0) = 6 > 0 \Rightarrow f(1,0) = -2$ is a local min

$D(-1,0) = 36 > 0$ and $f_{xx}(-1,0) = -6 < 0 \Rightarrow f(-1,0) = 2$ is local max

## Absolute Extrema

The absolute extrema exist if $f(x,y)$ is continuous on a closed and bounded set.




Steps: 1. Find the critical points.

2. Plug those critical points into $f$

3. Plug boundary points into $f$

4. Pick out the abs. max and abs. min.

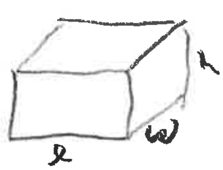
Optimization:

$$x + y + z = 100 \Rightarrow z = 100 - x - y$$

$$P = xyz = xy(100 - x - y)$$

$$\frac{\partial P}{\partial x} = 0$$

$$\frac{\partial P}{\partial y} = 0$$




$$V = lwh$$

$$S = 2lh + 2wh + lw \Rightarrow h = \frac{S - lw}{2(l+w)}$$

$$V(l,w)$$

$$\frac{\partial V}{\partial l} = 0$$

$$\frac{\partial V}{\partial w} = 0$$