Wed 10/8

Test II Review

Test II is Friday 10/10 covering §14.1 – 14.6. A review sheet is on Canvas.

There will be 12 5-point questions, mostly multiple choice, and 4 10-point questions, like quizzes where you have to show all work.

Office Hours: Today 5:30 – 6:30

Tomorrow 3:00 – 6:00

Ayres Hall 236

Zoom (link is in the syllabus)

May have to send an email to let me know you are Zoom.

(14) $f_y(a,b) = \lim_{y \to b} \dfrac{f(a,y) - f(a,b)}{y-b}$

True False

You need to know the definitions of f_x and f_y.

$$f_y(a,b) = \lim_{h \to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

Let $y = b+h$. Then $h = y-b$. Substituting, we have

$$f_y(a,b) = \lim_{y \to b} \frac{f(a,y) - f(a,b)}{y-b}$$

(15) $f_{xy} = \dfrac{\partial^2 f}{\partial x \partial y}$ $\Rightarrow$ $(f_x)_y = \dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right) = \dfrac{\partial^2 f}{\partial y \partial x} \neq \dfrac{\partial^2 f}{\partial x \partial y} = f_{yx}$

True **False**

(12) But from Clairaut's theorem, most of the functions we have $f_{xy} = f_{yx}$.

If f is defined on a disk D that contains (a,b) and f_{xy}, f_{yx} are continuous on D, then

$$f_{xy}(a,b) = f_{yx}(a,b).$$

An example of this not being true is

$$f(x,y) = \begin{cases} xy\,\dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

It can be shown $\displaystyle\lim_{(x,y)\to(0,0)} xy\,\frac{x^2-y^2}{x^2+y^2} = 0$

This is a removable discontinuity. However,

$$f_{xy}(0,0) \neq f_{yx}(0,0)$$

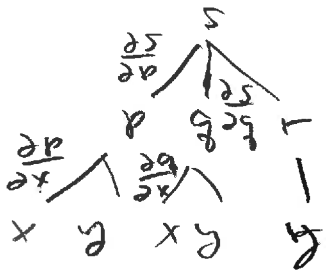
(9) $\displaystyle\lim_{(x,g(x))\to(a,b)} f(x,y) = L_1 \neq L_2 = \lim_{(x,h(x))\to(a,b)} f(x,y)$

Limit does not exist

Chain Rule:

$s = f(p,q,r)$, $p = g(x,y)$, $q = h(x,y)$, $r = l(y)$

$$\frac{\partial s}{\partial x} = \frac{\partial s}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial s}{\partial q}\frac{\partial q}{\partial x}$$

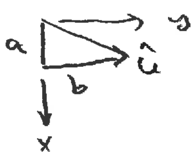


$$\frac{\partial s}{\partial y} = \frac{\partial s}{\partial p}\frac{\partial p}{\partial y} + \frac{\partial s}{\partial q}\frac{\partial q}{\partial y} + \frac{\partial s}{\partial r}\frac{\partial r}{\partial y}$$

(30) Definition of directional derivative.

$$D_{\hat{u}} f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb) - f(x_0, y_0)}{h}$$

$\hat{u} = \langle a, b \rangle$ $\quad a^2 + b^2 = 1$



what if $a = 1$ and $b = 0$, $\hat{u} = \langle 1, 0 \rangle = \hat{u}$

$$D_{\hat{u}} f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h} = f_x(a,b)$$

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$D_{\hat{u}} f = \nabla f \cdot \hat{u} = a\frac{\partial f}{\partial x} + b\frac{\partial f}{\partial y} = |\nabla f||\hat{u}|\cos\theta$$

$$= |\nabla f| \cos\theta$$

$$-|\nabla f| \le D_{\hat{u}} f \le |\nabla f|$$

(2) $f(x,y) = \dfrac{1}{3x^2+10y^2}$

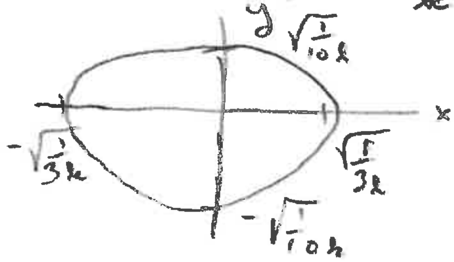
domain is $3x^2+10y^2 \neq 0 \Rightarrow 3x^2 \neq -10y^2$ an impossibility, so the only point $(0,0)$.

range $3x^2+10y^2 \geq 0 \Rightarrow \dfrac{1}{3x^2+10y^2} \geq 0$

but $(0,0)$ not in domain, range $= (0,\infty)$

level curves $\dfrac{1}{3x^2+10y^2} = k$ is a constant, Then

$3x^2+10y^2 = \frac{1}{k} = c$ a constant

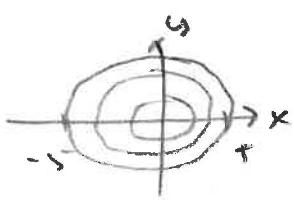


an elipse.

(3) $f(x,y) = \sqrt{25-x^2-y^2}$

domain : $25-x^2-y^2 \geq 0 \Rightarrow x^2+y^2 \leq 5^2$

Domain $= \{(x,y) \mid x^2+y^2 \leq 25\}$



range : $[0,5]$

$f(0,0) = \sqrt{25} = 5$

$f(5,0) = \sqrt{25-25} = 0$

(33) $xy^2z^3 = 8$ at $(2,2,1)$ find eqs. of tangent plane and normal line.

$F(x,y,z) = xy^2z^3 = 8$ level curve

$\nabla F = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

$\nabla F(2,2,1) = \langle 4, 8, 24 \rangle$ is normal to level surface, normal to tangent place.

Tangent place eq is

$$4x + 8y + 24z = 4(2) + 8(2) + 24(1) = 48 \Rightarrow$$

$$x + 2y + 6z = 12$$

Normal line is

$$\left.\begin{aligned} x &= 2 + 4t \\ y &= 2 + 8t \\ z &= 1 + 24t \end{aligned}\right\} \Rightarrow \frac{x-2}{4} = \frac{y-2}{8} = \frac{z-1}{24} \Rightarrow$$

$$\frac{1}{4}\,\frac{x-2}{1} = \frac{1}{4}\,\frac{y-2}{2} = \frac{1}{4}\,\frac{z-1}{6} \Rightarrow$$

$$\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-1}{6}$$