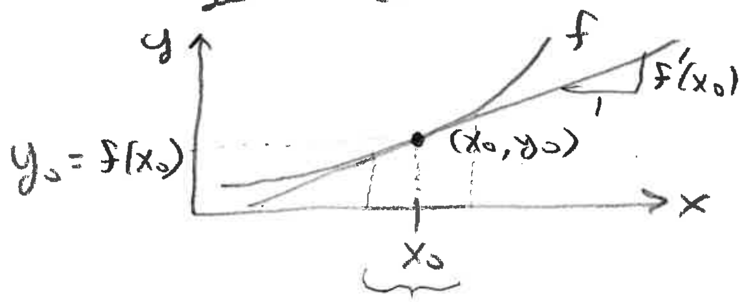
Tues 9/23

§14.4 Tangent Planes & Linear Approximations

Recall from Calculus I how to find the eq. of a tangent line.



$y = f(x)$

The point-slope form of the tangent line is

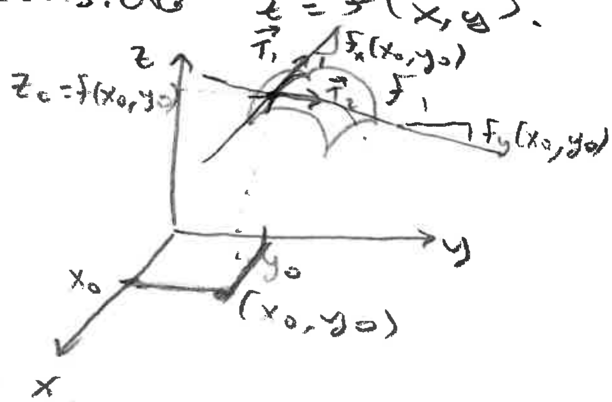
$$y = y_0 + f'(x_0)(x - x_0)$$

Recall to find the eq. of a plane, we need a point (x_0, y_0, z_0) and a vector $\langle a, b, c\rangle$ normal to the plane. The eq. of the plane is

$$ax + by + cz = ax_0 + by_0 + cz_0 \Rightarrow$$

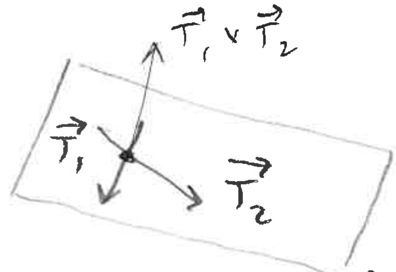
$$z = z_0 - \frac{a}{c}(x - x_0) - \frac{b}{c}(y - y_0)$$

Consider $z = f(x, y)$.



$$\vec{T}_1 = \hat{i} + f_x(x_0, y_0)\hat{k}$$

$$\vec{T}_2 = \hat{j} + f_y(x_0, y_0)\hat{k}$$



(x_0, y_0, z_0)

$$\vec{T}_1 \times \vec{T}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & f_x(x_0,y_0) \\ 0 & 1 & f_y(x_0,y_0) \end{vmatrix}$$

$$= \hat{i}\,(0 - f_x(x_0,y_0)) - \hat{j}\,(f_y(x_0,y_0) - 0) + \hat{k}\,(1-0)$$

$$= \langle -f_x(x_0,y_0), -f_y(x_0,y_0), 1 \rangle$$

The equation of the tangent plane is

$$-f_x(x_0,y_0)x - f_y(x_0,y_0)y + 1z = -f_x(x_0,y_0)x_0 - f_y(x_0,y_0)y_0 + z_0$$

which can be rewritten as

$$z = z_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Example: Find the equation of the plane that is tangent to the curve $z = 2x^2 + y^2 - 5y$ at the point $(1, 2, -4)$.

Solution: $x_0 = 1$, $y_0 = 2$, $z_0 = -4$. Let $z = f(x,y)$

$f_x = 4x$ $\qquad f_x(1,2) = 4(1) = 4$

$f_y = 2y - 5$ $\qquad f_y(1,2) = 2(2) - 5 = -1$

$$z = -4 + 4(x-1) - 1(y-2)$$

$$= 4x - y - 6$$

$z(1.1, 1.9) = 4(1.1) - 1.9 - 6 = 4.4 - 1.9 - 6 = -3.5$

$f(1.1, 1.9) = 2(1.1)^2 + (1.9)^2 - 5(1.9) = -3.47$