Fri 9/19

Example: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}}$

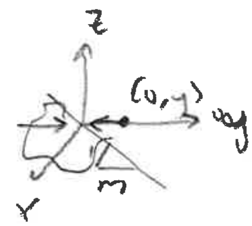
Step 1 yields $\frac{0}{0}$ discontinuous at $(0,0)$

Step 2 can't rewrite

Step 3 Find a jump discontinuity

Let $x=0$.

$\displaystyle\lim_{(0,y)\to(0,0)} \frac{0}{\sqrt{y^2}} = 0$



Let $y=mx$, where m is a constant.

$\displaystyle\lim_{(x,mx)\to(0,0)} \frac{2x(mx)}{\sqrt{x^2+m^2x^2}} = \frac{2m}{\sqrt{1+m^2}} \lim_{x\to 0} \frac{x^2}{|x|}$

$\displaystyle = \frac{2m}{\sqrt{1+m^2}} \lim_{x\to 0} \frac{|x|^2}{|x|} = \frac{2m}{\sqrt{1+m^2}} \lim_{x\to 0} |x|$

$= 0$

Step 4 Use the definition!

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(\vec{x})$ exists and $0 < |\vec{x}-\vec{a}| < \delta$, we have $|f(\vec{x}) - L| < \varepsilon$. Then $\displaystyle\lim_{x\to\vec{a}} f(\vec{x}) = L$

I believe $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}} = 0 = L$.

Choose $\varepsilon > 0$. Let $\delta = \frac{\varepsilon}{2}$. Then for

$0 < |\vec{x}-\vec{a}| = |\langle x,y\rangle - \langle 0,0\rangle| = \sqrt{x^2+y^2} < \delta$, we have

$\displaystyle |f(\vec{x}) - L| = \left| \frac{2xy}{\sqrt{x^2+y^2}} - 0 \right| = \frac{2|x||y|}{\sqrt{x^2+y^2}} = 2\left(\frac{\sqrt{x^2}}{\sqrt{x^2+y^2}}\right)\sqrt{y^2}$

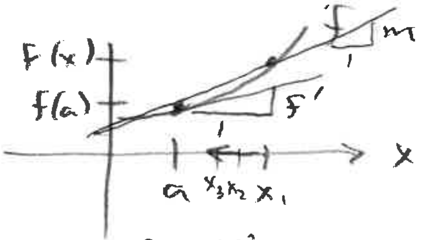
$\le 2\sqrt{y^2} \le 2\sqrt{x^2+y^2} < 2\delta = 2\frac{\varepsilon}{2} = \varepsilon$

§14.3 Partial Derivatives

We calculate the partial derivative of a function $f(\vec{x})$, which is a multivariable function, by differentiating with respect to one of the variable while holding the other variables constant.

Recall the definition of a derivative from Calculus I.

$$f'(x) = \frac{df}{dx} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



$\{x_i\}_{i=1}^{\infty} \to a$

$\{m_i\}_{i=1}^{\infty} \to \frac{df}{dx}$

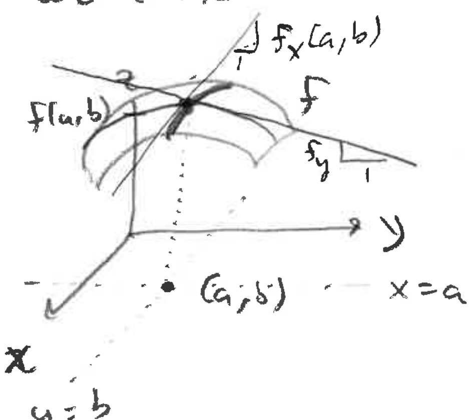
The same thing applies in partial derivatives. We use the notation $\frac{\partial f}{\partial x}$. The definition is

$$\frac{\partial f}{\partial x} = f_x = D_{\hat{i}} f = \lim_{\vec{x} \to \vec{a}} \frac{f(\vec{x}) - f(\vec{a})}{x - a}$$

where $\vec{x} = \langle x, y, z \rangle$ and $\vec{a} = \langle a, b, c \rangle$

Suppose $f(x,y)$, then at (a,b), we have

$$f_x(a,b) = \lim_{x \to a} \frac{f(x,b) - f(a,b)}{x - a}$$



Let $h = x - a$. Then $x = a + h$.

$$f_x(a,b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a,b)}{h}$$

In general,

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(\vec{x} + h\hat{i}) - f(\vec{x})}{h}$$

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(\vec{x} + h\hat{j}) - f(\vec{x})}{h}$$

Example: $f(x,y) = x^4 + 5xy^3$

$$\frac{\partial f}{\partial x} = 4x^3 + 5y^3$$

$$\frac{\partial f}{\partial y} = 15xy^2$$

Example: $g(u,v) = (u^2v - v^3)^5$

$$\frac{\partial g}{\partial u} = 5(u^2v - v^3)^4(2uv) = 10uv(u^2v - v^3)^4$$

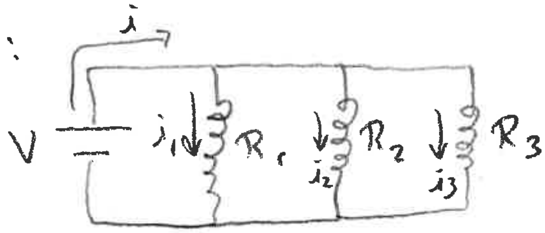
$$\frac{\partial g}{\partial v} = 5(u^2v - v^3)^4(u^2 - 3v^2)$$

Example: $R(s,t) = te^{s/t}$. Find $R_t(0,1)$.

$$R_t = e^{s/t} + te^{s/t}\left(-\frac{s}{t^2}\right) = \left(1 - \frac{s}{t}\right)e^{s/t}$$

$$R_t(0,1) = \left(1 - \frac{0}{1}\right)e^{0/1} = 1$$

Example:



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$\frac{\partial R}{\partial R_1} = -\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-2}\left(-\frac{1}{R_1^2}\right) = \frac{R^2}{R_1^2}$$