

Tues 9/16

Two-Dimensional Functions

Example: $h(x,y) = e^{\sqrt{y-x^2}}$

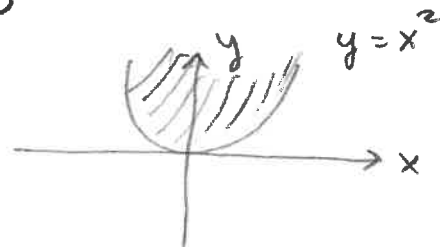
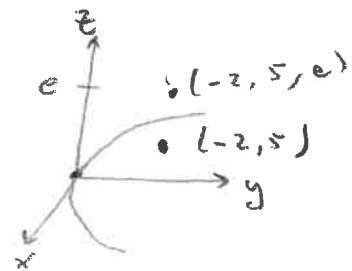
$$h(-2,5) = e^{\sqrt{5-(-2)^2}} = e$$

$$D = \{(x,y) \mid y-x^2 \geq 0\}$$

$$y-x^2 \geq 0 \Rightarrow y \geq x^2$$

$$\text{range} = [1, \infty)$$

$$h(0,0) = e^{\sqrt{0-0}} = e^0 = 1$$



Example: $g(x,y) = \sin^{-1}(x+y)$

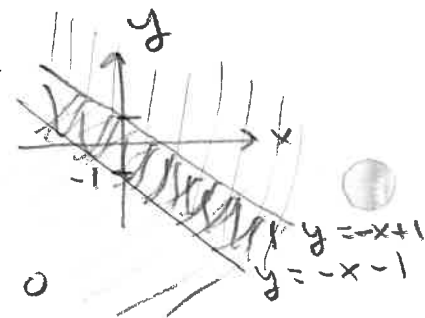
$$D = \{(x,y) \mid -1 \leq x+y \leq 1\}$$

$$g(0,0) = \sin^{-1}(0+0) = \sin^{-1}0 = 0$$

$$-1 \leq x+y \Rightarrow y \geq -x-1$$

$$x+y \leq 1 \Rightarrow y \leq -x+1$$

$$\text{range} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$



We can also indicate 2D functions with a table.

$W = f(T, v)$ is wind chill a function of temp T and wind speed v .

$v \backslash T$	5	10	15
5	4	3	2
0	-2	-3	-4
-5	-7	-9	-11

$$W = f(5, 15) = 2$$

$$W = f(0, v) = -3 \Rightarrow v = 10$$

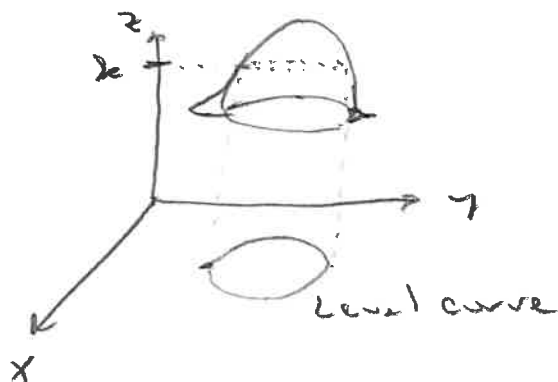
$$W = f(5, v) = \frac{-1}{5}v + 5$$

Level curves

Let k be a constant in the range of $f(x,y)$.

A level curve is the set of all points such that

$$f(x,y) = k.$$



Example: $f(x,y) = x^2 - y^2$

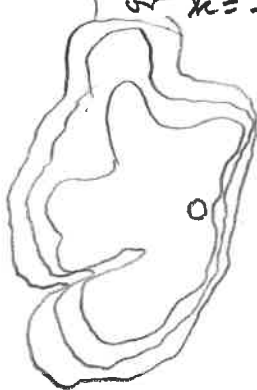
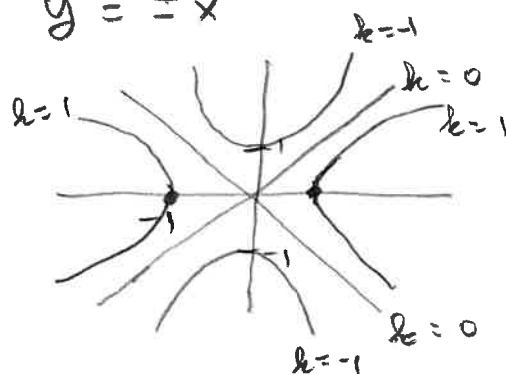
Find level curves for $k = 0, \pm 1$. Graph them.

For $k=0$, $x^2 - y^2 = 0 \Rightarrow y = \pm x$

For $k=1$, $x^2 - y^2 = 1 \Rightarrow$

$$y = \pm \sqrt{x^2 - 1}$$

For $k=-1$, $x = \pm \sqrt{y^2 - 1}$



Another example of a level curve
is a contour map.

Three-Dimensional Functions

Example: Let x = # small boxes for \$2.00

y = # med boxes for \$3.50.

z = # large boxes for \$4.00.

Fixed cost is \$1000. Write the cost function C .

$$C(x,y,z) = 2x + 3.5y + 4z + 1000$$