Mon 9/15

Quiz #4 Tuesday 9/16 over curvature.

Find the curvature for $\vec{r}(t) = \langle t, t^2, e^t \rangle$ at the point $(0,0,1)$.

Method 1:

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$\vec{r}'(t) = \langle 1, 2t, e^t \rangle$$

$$|\vec{r}'(t)| = \sqrt{1+4t^2+e^{2t}}$$

$$\hat{T}(t) = \frac{\langle 1, 2t, e^t \rangle}{\sqrt{1+4t^2+e^{2t}}} = \langle 1, 2t, e^t \rangle (1+4t^2+e^{2t})^{-1/2}$$

$$\vec{T}'(t) = \langle 0, 2, e^t \rangle (1+4t^2+e^{2t})^{-1/2} - \frac{1}{2}(1+4t^2+e^{2t})^{-3/2}(8t+2e^{2t})\langle 1, 2t, e^t \rangle$$

$t = 0$ at $(0,0,1)$

$$\vec{T}'(0) = \langle 0, 2, 1 \rangle (1+1)^{-1/2} - \frac{1}{2}(1+1)^{-3/2}(2)\langle 1, 0, 1 \rangle$$

$$= \frac{\langle 0,2,1 \rangle}{\sqrt{2}} - \frac{\langle 1,0,1 \rangle}{2\sqrt{2}} = \frac{\langle 0,4,2 \rangle - \langle 1,0,1 \rangle}{2\sqrt{2}}$$

$$= \frac{\langle -1, 4, 1 \rangle}{2\sqrt{2}}$$

$$|\vec{T}'(0)| = \frac{\sqrt{1+16+1}}{2\sqrt{2}} = \frac{1}{2}\sqrt{\frac{18}{2}} = \frac{3}{2}$$

$$|\vec{r}'(0)| = \sqrt{2}$$

$$\kappa(0) = \frac{3/2}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$$

Method 2:

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

$\vec{r}\,'(t) = \langle 1, 2t, e^t \rangle$

$\vec{r}\,''(t) = \langle 0, 2, e^t \rangle$

$t = 0$ at $(0, 0, 1)$

$\vec{r}\,'(0) = \langle 1, 0, 1 \rangle$

$\vec{r}\,''(0) = \langle 0, 2, 1 \rangle$

$$\vec{r}\,'(0) \times \vec{r}\,''(0) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \hat{i}(0-2) - \hat{j}(1-0) + \hat{k}(2-0)$$

$$= \langle -2, -1, 2 \rangle$$

$|\vec{r}\,'(0) \times \vec{r}\,''(0)| = \sqrt{4+1+4} = 3$

$|\vec{r}\,'(0)| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$

$$\kappa(0) = \frac{3}{(\sqrt{2})^3} = \frac{3}{2\sqrt{2}}$$

§14.1 Multidimensional Functions

In Chapter 13, we look at $\vec{r} : \mathbb{R} \to \mathbb{R}^3$. Now we switch this. In Chapters 14 & 15, we have

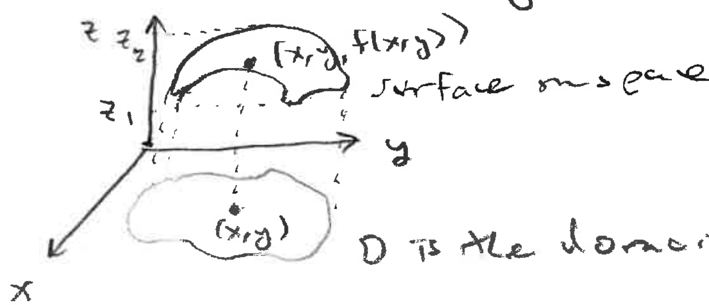
$$f : \mathbb{R}^3 \to \mathbb{R}$$

we have a member of the domain D of the form

$$\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$$

Two-dimensional Functions

We write $z = f(x,y)$



D is the domain, a region in $\mathbb{R}^2$

range is $[z_1, z_2]$.

Example: $g(x,y) = x^2 \ln(x+y)$

Evaluate g at $(3,1)$.

Find the domain.

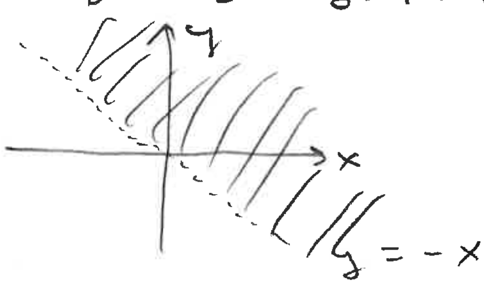
Graph the domain.

Find the range.

Solution: $g(3,1) = 3^2 \ln(3+1) = 9\ln 4$

Domain requires that $x+y > 0$

$D = \{(x,y) \mid x+y > 0\}$.



$x+y>0 \Rightarrow y > -x$

range is $(-\infty, \infty)$

$g(e^{-1}, 0) = e^{-2} \ln e^{-1} = -\frac{1}{e^2}$