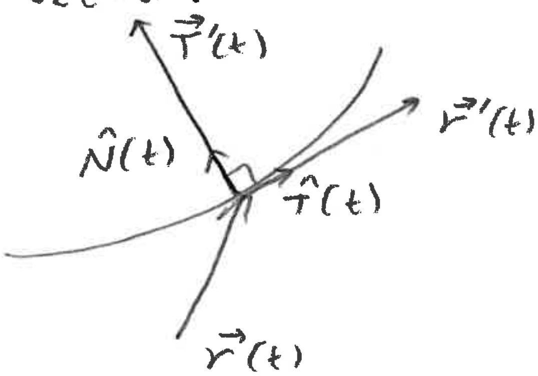
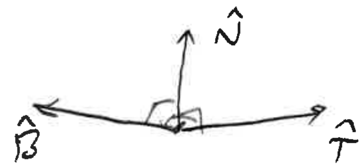
$$|\vec{r}'(1) \times \vec{r}''(1)| = \sqrt{36+36+4}$$

$$k = \frac{\sqrt{76}}{|\langle 1,2,3\rangle|^3} = \frac{\sqrt{76}}{(\sqrt{14})^3} = \frac{2\sqrt{19}}{14\sqrt{14}} = \frac{1}{7}\sqrt{\frac{19}{14}}$$

TNB frame: $\hat{B} = \hat{T} \times \hat{N}$ called binormal unit vector.





Wed 9/10

Test 1 Friday §12.1 - 12.5, 13.1 - 13.2

12 m/c questions worth 5 pts each (60 pts).

4 quiz-type questions worth 10 pts each (40 pts.)

(12.13) this question will not be on the test. However, you will need to find a vector between two points.

$\vec{a} = \overline{PQ} = \langle 1-7, -5+5, 2-5\rangle = \langle -6, 0, -3\rangle$

$\vec{b} = \overline{PS} = \langle -5, 8, -5\rangle$

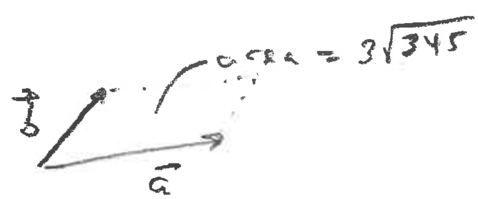
Find the cross product

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & -3 \\ -5 & 8 & -5 \end{vmatrix} = \hat{i}\,(0+24) - \hat{j}\,(30-15) + \hat{k}\,(-48-0)$$

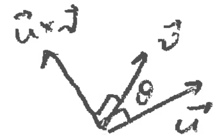
$$= 24\hat{i} - 15\hat{j} - 48\hat{k}$$

Find the length of a vector.

$$|\vec{a}\times\vec{b}| = \sqrt{24^2+15^2+48^2}$$



You need to know



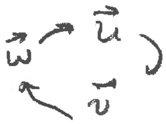
$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$

$(\vec{u}\times\vec{v})\cdot\vec{u} = 0 = (\vec{u}\times\vec{v})\cdot\vec{v}$

Triple scalar product

$(\vec{u}\times\vec{v})\cdot\vec{w} = \vec{v}\cdot(\vec{w}\times\vec{u}) = (\vec{w}\times\vec{u})\cdot\vec{v}$

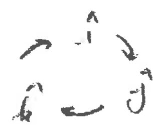


$\vec{u}\times\vec{v} \neq \vec{v}\times\vec{u}$ $\qquad \vec{u}\times\vec{v} = -(\vec{v}\times\vec{u})$

$|\vec{u}\times\vec{v}| = |\vec{v}\times\vec{u}|$

$c(\vec{u}\cdot\vec{v}) = c\vec{u}\cdot\vec{v} = \vec{u}\cdot(c\vec{v})$

$\hat{i}\times\hat{j} = \hat{k}$ $\qquad \hat{k}\times\hat{j} = -\hat{i}$

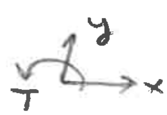


$\hat{k}\times\hat{i} = \hat{j}$

$$\hat{k}\times\hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-1) + \hat{k}(0-0)$$

$$= \hat{j}$$

(12.15) $\vec{\tau} = \vec{r}\times\vec{F}$



$\vec{r} = 5\hat{i}$

$\vec{F} = 20\frac{\sqrt{2}}{2}\hat{i} - 20\frac{\sqrt{2}}{2}\hat{j}$

$\vec{\tau} = \vec{r}\times\vec{F} = -50\sqrt{2}\,\hat{k}$

$|\vec{\tau}| = 50\sqrt{2}\ \cancel{\text{in}}\cdot\text{lb}\left(\frac{1\text{ ft}}{12\cancel{\text{in}}}\right)$ $\qquad 1 = \frac{12\text{ in}}{12\text{ in}} = \frac{1\text{ ft}}{12\text{ in}}$

$= \frac{25}{6}\sqrt{2}\text{ ft}\cdot\text{lb}$

241-002 (21)

(12.26) $$\frac{2(4)+2(-6)+8-(-6)}{\sqrt{4+4+1}} = \frac{10}{\sqrt{9}} = \frac{10}{3}$$

(13.8) $$\vec{r}\,'(t) = \hat{\imath} + (4t^3 - 10t)\hat{\jmath} + \frac{1}{t+2}\hat{k}$$

$$\vec{r}(t) = \int \vec{r}\,'(t)\,dt = \int dt\,\hat{\imath} + \int (4t^3-10t)\,dt\,\hat{\jmath} + \int \frac{1}{t+2}\,dt\,\hat{k}$$

$$= (t + C_1)\hat{\imath} + (t^4 - 5t^2 + C_2)\hat{\jmath} + (\ln(t+2) + C_3)\hat{k}$$

$$= t\,\hat{\imath} + (t^4 - 5t^2)\hat{\jmath} + \ln(t+2)\hat{k} + \vec{C}$$

$$\vec{r}(0) = \ln 2\,\hat{k} + \vec{C} = 2\hat{\jmath} + \ln 2\,\hat{k} \Rightarrow \vec{C} = 2\hat{\jmath}$$

$$\vec{r}(t) = t\,\hat{\imath} + (t^4 - 5t^2 + 2)\hat{\jmath} + \ln(t+2)\hat{k}$$