Mon 9/8

Test I Friday 9/12 covering sections
12.1 – 12.5, 13.1 – 13.2.

Quiz 3 Problem is Tuesday 9/9.

Find the parametric equations of a line tangent to $\vec{r}(t) = (t^2+1)\,\hat{i} + 4\sqrt{t}\,\hat{j} + e^{t^2-t}\,\hat{k}$

at the point $(2,4,1)$. Also, find the unit tangent vector at the given point.

1 Name First Last

3 $\vec{r}\,'(t) = 2t\,\hat{i} + \dfrac{2}{\sqrt{t}}\,\hat{j} + (2t-1)\,e^{t^2-t}\,\hat{k}$

$$\vec{r}(t) = \langle 2,4,1\rangle \Rightarrow \left.\begin{array}{l} t^2+1 = 2 \\ 4\sqrt{t} = 4 \\ e^{t^2-t} = 1 \end{array}\right\} \Rightarrow t = 1$$

1 $\vec{r}\,'(1) = \langle 2,2,1\rangle$

3
$x = 2 + 2t$

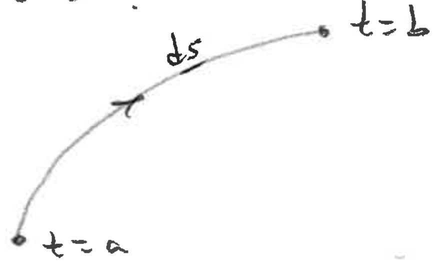
$y = 4 + 2t$

$z = 1 + t$

2 $$\hat{T}(1) = \frac{\vec{r}\,'(1)}{|\vec{r}\,'(1)|} = \frac{\langle 2,2,1\rangle}{\sqrt{4+4+1}} = \left\langle \tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3}\right\rangle$$

§13.3 Arc Length & Curvature

Suppose we have a curve that is parametrized by $a \le t \le b$.



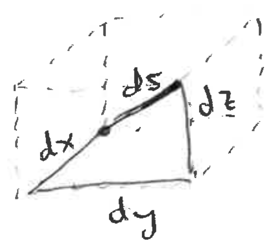
Let s be the distance along the curve, increasing with increasing t.

$s(a) = 0$ initial point and $s(b) = L$ the length of the curve. Note that

$$\frac{ds}{dt} = |\vec{r}\,'(t)| > 0$$

velocity speed direction

$$\vec{r}\,'(t) = |\vec{r}\,'(t)|\ \hat{T}(t)$$



$$ds = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

$$= |\vec{r}\,'(t)|\ dt.$$

$\vec{r}(t) = \langle x, y, z \rangle$

$\vec{r}\,'(t) = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$

The arc length is then

$$L = s(b) - s(a) = \int_{s(a)}^{s(b)} ds = \int_a^b |\vec{r}\,'(t)|\ dt$$

Example: Find the length of the curve for

$\vec{r}(t) = \sqrt{2}\, t\, \hat{i} + e^{t}\hat{j} + e^{-t}\hat{k}$, $0 \le t \le 1$

Solution: $L = \int_0^1 |\vec{r}\,'(t)|\, dt$

$\vec{r}\,'(t) = \sqrt{2}\,\hat{i} + e^{t}\hat{j} - e^{-t}\hat{k}$

$|\vec{r}\,'(t)| = \sqrt{2 + e^{2t} + e^{-2t}}$

$L = \int_0^1 \sqrt{2 + e^{2t} + e^{-2t}}\, dt = \int_0^1 \sqrt{(e^{t} + e^{-t})^2}\, dt$

$= \int_0^1 (e^{t} + e^{-t})\, dt = e^{t} - e^{-t} \Big|_0^1$

$= e^{1} - e^{-1} - (e^{0} - e^{0}) = e - \frac{1}{e}$

Example: Find the arc length function $s(t)$ for

$\vec{r}(t) = (5-t)\hat{i} + (4t-3)\hat{j} + 3t\,\hat{k}$

measured from the point $(4,1,3)$, reparametrize the curve with respect to arc length s, and find $\vec{r}(s=4)$.

Solution: $\vec{r}\,'(t) = -\hat{i} + 4\hat{j} + 3\hat{k}$

$|\vec{r}\,'(t)| = \sqrt{1+16+9} = \sqrt{26}$

$s(t) = \int_1^t \sqrt{26}\, dt = \sqrt{26}\,(t-1) \Rightarrow t = \frac{s}{\sqrt{26}} + 1$

$\vec{r}(s) = \left(4 - \frac{s}{\sqrt{26}}\right)\hat{i} + \left(\frac{4s}{\sqrt{26}} + 1\right)\hat{j} + \left(\frac{3s}{\sqrt{26}} + 3\right)\hat{k}$

$\vec{r}(4) = \left(4 - \frac{4}{\sqrt{26}}\right)\hat{i} + \left(\frac{16}{\sqrt{26}} + 1\right)\hat{j} + \left(\frac{12}{\sqrt{26}} + 3\right)\hat{k}$