$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \hat{i}(2-2) - \hat{j}(2-1) + \hat{k}(2-1)$$

$$= \langle 0, -1, 1 \rangle$$

$$\vec{r} = \langle 1, 0, 0 \rangle + t\langle 0, -1, 1 \rangle = \langle 1, -t, t \rangle$$

Wed 10/3

Example: Given a line $x = 2 - 2t$, $y = 3t$, $z = 1 + t$ and a plane $x + 2y - z = 7$. Find the point of intersection.

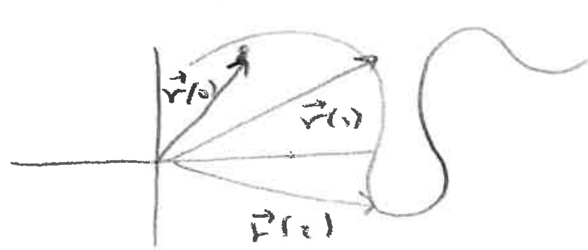
Solution: Four eqs and four unknowns can be solved by substitution.

$$2 - 2t + 2(3t) - (1 + t) = 7 \Rightarrow 3t = 6 \Rightarrow$$
$$t = 2$$
$$(2 - 2(2), 3(2), 1 + 2) = (-2, 6, 3)$$

§13.1 Vector Valued Functions

$$\vec{r}(t) = \langle x, y, z \rangle = \langle f(t), g(t), h(t) \rangle$$

This is a vector-valued function.



The domain of $\vec{r}$ is the intersection of the domains of f, g, and h.

Example: $\vec{r}(t) = \langle \ln(t+2), \sqrt{t-3}, \frac{1}{t-4} \rangle$.

Find the domain.

Solution: $f(t) = \ln(t+2)$ has domain $(-2, \infty)$

$t+2 > 0 \Rightarrow t > -2$

$g(t) = \sqrt{t-3}$ has domain $[3, \infty)$

$t - 3 \geq 0 \Rightarrow t \geq 3$

So far $(-2, \infty) \cap [3, \infty) = [3, \infty)$

$h(t) = \frac{1}{t-4}$ has domain $(-\infty, 4) \cup (4, \infty)$

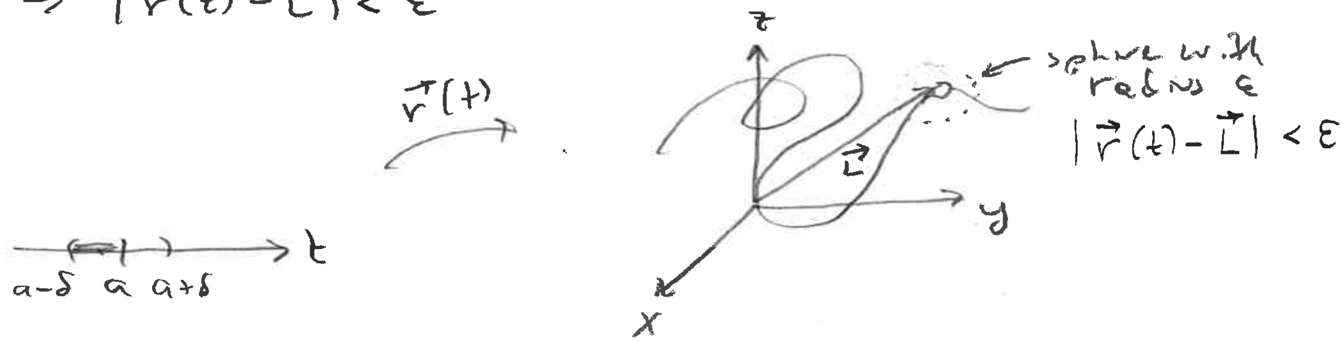
The domain of $\vec{r}$ is $[3, 4) \cup (4, \infty)$

Limit of vector-valued Function

$$\lim_{t \to a} \vec{r}(t) = \lim_{t \to a} \langle f(t), g(t), h(t) \rangle = \langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle$$

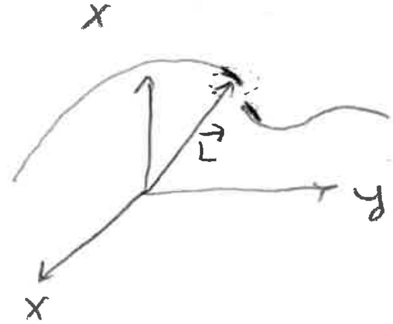
Formal Definition of a Limit:

For every $\varepsilon > 0$ there exists $\delta > 0$ such that $|t - a| < \delta$
$\Rightarrow |\vec{r}(t) - \vec{L}| < \varepsilon$



$\vec{r}(a) = \lim_{t \to a} \vec{r}(t)$

we say $\vec{r}$ is continuous at $t = a$.



Example: $\lim_{t\to 1}\left(\frac{t^2-1}{t-1}\hat{\imath} + \sqrt{t+8}\,\hat{\jmath} + \frac{\sin \pi t}{\ln t}\hat{k}\right)$

$$= \lim_{t\to 1}\frac{t^2-1}{t-1}\hat{\imath} + \lim_{t\to 1}\sqrt{t+8}\,\hat{\jmath} + \lim_{t\to 1}\frac{\sin \pi t}{\ln t}\hat{k}$$

$$= \lim_{t\to 1}\frac{(t+1)(t-1)}{t-1}\hat{\imath} + 3\hat{\jmath} + \lim_{t\to 1}\frac{\pi \cos \pi t}{1/t}\hat{k}$$

$$= 2\hat{\imath} + 3\hat{\jmath} - \pi\hat{k}$$

Space Curves

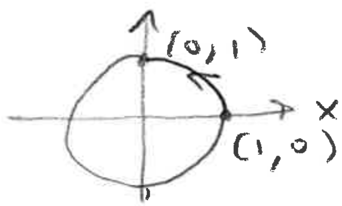
Consider $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$.

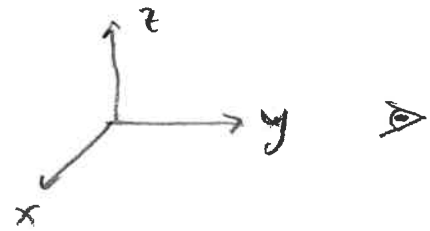
It is sometimes helpful to graph the projection onto one of the planes x-y, x-z, or y-z.

Projection onto the x-y plane.

$x = \cos t$ $\qquad x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

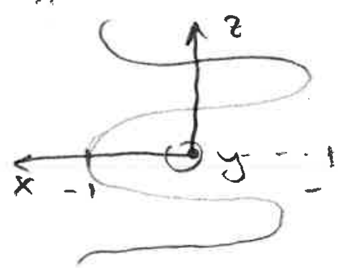
$y = \sin t$





Projection onto the x-z plane.

$$\left.\begin{array}{l} x = \cos t \\ z = t \end{array}\right\} \Rightarrow x = \cos z$$

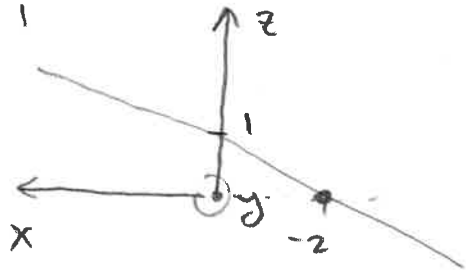


Example: Find an equation of the plane containing the space curve $\vec{r}(t) = \langle 2t, t^2, t+1 \rangle$.

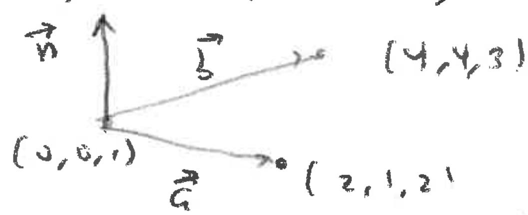
Solution: $x = 2t$, $y = t^2$, $z = t+1$

$t = z - 1$

$\boxed{x = 2(z-1) = 2z - 2}$



The points $(0,0,1)$, $(2,1,2)$, $(4,4,3)$.



$\vec{a} = \langle 2, 1, 1 \rangle$

$\vec{b} = \langle 4, 4, 2 \rangle$

$$\hat{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 4 & 4 & 2 \end{vmatrix} = \hat{i}(2-4) - \hat{j}(4-4) + \hat{k}(8-4)$$

$$= \langle -2, 0, 4 \rangle$$

$$-2x + 0y + 4z = 4 \Rightarrow -x + 2z = 2 \Rightarrow$$

$$-x = -2z + 2 \Rightarrow \boxed{x = 2z - 2}$$

Example: $\vec{r}_1 = \langle t^2, 7t-12, t^2 \rangle$ be an aircraft.

$\vec{r}_2 = \langle 4t-3, t^2, 5t-6 \rangle$ be a missile.

$\vec{r}_1(a) = \vec{r}_2(a)$

$t^2 = 4t - 3 \Rightarrow t^2 - 4t + 3 = (t-3)(t-1) = 0 \Rightarrow t = 1, 3$

$\vec{r}_1(1) = \langle 1, -5, 1 \rangle \neq \vec{r}_2(1) = \langle 1, 1, -1 \rangle$

$\vec{r}_1(3) = \langle 9, 9, 9 \rangle = \vec{r}_2(3) = \langle 9, 9, 9 \rangle$