Tues 9/2

§12.5 Lines and Planes

Distance from a Point to a Plane

Last time, we saw that the eq of a plane is

$$ax+by+cz+d=0$$

where $\vec{n}=\langle a,b,c\rangle$. If (x_0,y_0,z_0) is in the plane, then

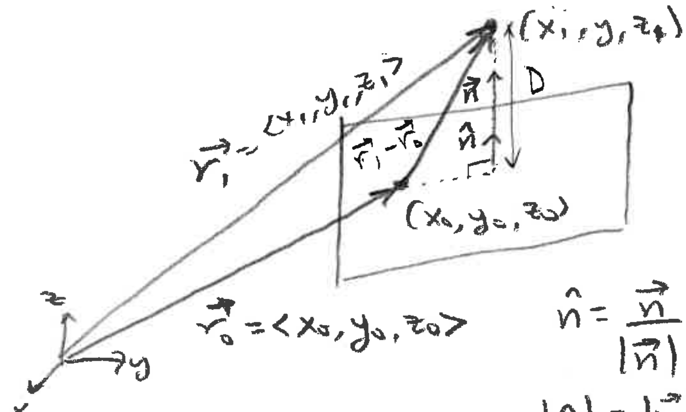
$$ax_0+by_0+cz_0+d=0 \Rightarrow$$

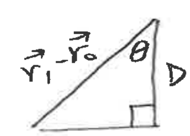
$$d=-(ax_0+by_0+cz_0)$$

Suppose (x_1,y_1,z_1) is not in the plane. Then

$$ax_1+by_1+cz_1+d \neq 0.$$

Let error $=|ax_1+by_1+cz_1+d|$. We might suppose that the error is related to the distance.





$$\hat{n}=\frac{\vec{n}}{|\vec{n}|}$$

$$|\hat{n}|=\frac{|\vec{n}|}{|\vec{n}|}=1$$

$$D=|\vec{r}_1-\vec{r}_0||\hat{n}||\cos\theta|$$

$$=|(\vec{r}_1-\vec{r}_0)\cdot\hat{n}|$$

So the distance from the point to the plane is

$$D=|\vec{r}_1\cdot\hat{n}-\vec{r}_0\cdot\hat{n}|=\frac{|ax_1+by_1+cz_1-(ax_0+by_0+cz_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

Example: Find the distance from $(1,-2,4)$ to $3x+2y+6z=5$.

Solution: $D = \dfrac{|3(1)+2(-2)+6(4)-5|}{\sqrt{9+4+36}} = \dfrac{|23-5|}{\sqrt{49}} = \dfrac{18}{7}$

Example: Find the distance between the parallel planes $2x-3y+z=4$ and $4x-6y+2z=3$.

Solution: $\vec{n}_1 = \langle 2,-3,1\rangle$ and $\vec{n}_2 = \langle 4,-6,2\rangle = 2\vec{n}_1$

So these planes are indeed parallel.

Note that $(2,0,0)$ is in the first plane. Then

$$D = \frac{|4(2)-6(0)+2(0)-3|}{\sqrt{16+36+4}} = \frac{|8-3|}{\sqrt{56}} = \frac{5}{\sqrt{56}}$$

$$= \frac{5}{2\sqrt{14}} = \frac{5\sqrt{14}}{28}$$

<u>Intersections</u>

Example: Find the line formed by the intersection of the planes $x+y+z=1$ and $x+2y+2z=1$.

Solution:
$$\left.\begin{array}{r} x+y+z=1 \\ x+2y+2z=1 \\ z=0 \end{array}\right\} \Rightarrow \left.\begin{array}{r} x+y=1 \\ x+2y=1 \end{array}\right\} \Rightarrow \begin{array}{l} x=1 \\ y=0 \end{array}$$

So the point $(1,0,0)$ is on the line.

Now we need $\vec{v}$, so we can write

$\vec{r} = \vec{r}_0 + \vec{v}t$ where $\vec{r}_0 = \langle 1,0,0\rangle$

Since $\vec{v}$ is in both planes, it is orthogonal to both $\vec{n}_1$ and $\vec{n}_2$.

$\vec{v} = \vec{n}_1 \times \vec{n}_2$

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \hat{i}(2-2) - \hat{j}(2-1) + \hat{k}(2-1)$$

$$= \langle 0, -1, 1 \rangle$$

$$\vec{r} = \langle 1, 0, 0 \rangle + t\langle 0, -1, 1 \rangle = \langle 1, -t, t \rangle$$